Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics: Section 1
Test 1

Name:____

Problem	Points	Score
1	30	
2	30	
3	20	
4	20	
Total	100	

Q1: (30pts)
In this question n DISTINGUISHABLE 6-sided dice are rolled sequentially.

(a) What is the probability that none of them come up 6?

(b) What is the expected number Z of distinct dice values that actually appear.

For example if $n = 5$ and the sequence of outcomes is 3,3,4,2,3 then $Z = 3$.

(c) What is the expected number of times N that the sequence 6,6,6 appears as a contiguous subsequence?

For example in the sequence 5,6,6,6,6,5,6,6,6,7,1,2,6 the value of N is 3.

Q2: (30pts)
m DISTINGUISHABLE balls are randomly placed in n distinct boxes.

(a) What is the probability that box 1 contains at most 1 ball?

(b) What is the expected number of boxes with 0 or 1 ball?

Q3: (20pts)
k INDISTINGUISHABLE balls are randomly coloured with n colours. What is the probability that each of the colours is used at least twice?

Q4: (30pts)
A particle starts at the origin and moves along the real line. At each step it moves 1 unit left with probability $1/2$ and moves 1 unit right with probability $1/2$.
What is the probability that it does NOT return to the origin at all within n steps?