21-301 Combinatorics
Homework 6
Due: Monday,November 10

1. There are n teams playing some sport. At some point in the season we have the following statistics: $w(i), i \in [n]$ denotes the number of games that team i has won so far. $p(i,j), i \neq j \in [n]$ denotes the number of times team i will play team j in the rest of the season. Set up the question: "Can team n still end up with the most wins?" as a network flow problem.

2. Find the set of P-positions for the take-away games with subtraction sets

   (a) $S = \{1, 3, 7\}$.

   (b) $S = \{1, 4, 6\}$.

   Suppose now that there are two piles and the rules for each pile are as above. Now find the P positions for the two pile game where in one pile S is as in (a) and the other pile is as in (b).

3. In a take-away game, the set S of the possible numbers of chips to remove is finite. Show that the Sprague-Grundy numbers satisfy $g(n) \leq |S|$ where n is the number of chips remaining.