(14) Let $F(t,x)$ define a $\mathscr{C}^1$ function and set

$$u(t,x) = \frac{1}{2}\int_0^t \int_{x-t+s}^{x+t-s} F(s,y)\,dy\,ds$$

show that $\dfrac{\partial^2 u}{\partial t^2} - \dfrac{\partial^2 u}{\partial x^2} = F$ (u solves a wave eqn.).

(15) Let $\vec{F}(x,y) = (2x + 2xy^2 e^{x^2y})\vec{i} + (3y^2 + (1+x^2y)e^{x^2y})\vec{j}$.

Let C be any curve in the plane starting from (a,b) and ending at (c,d).

Compute $\displaystyle\int_C \vec{F}\cdot d\vec{r}$

(16) 1) Let $R = \{(x,y,z) / x^2+y^2+z^2 \leq R_0^2\}$

$S = \{(x,y,z) / x^2+y^2+z^2 = R_0^2\}$

$$\vec{F}(x,y,z) = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{(x^2+y^2+z^2)^{3/2}}$$

show that $\displaystyle\iint_S \vec{F}\cdot d\vec{\sigma}_{out} = 4\pi$

2) What if S and R are such that $(0,0,0) \notin R$?

(17) Compute $\displaystyle\oint_C \vec{F}\cdot d\vec{r}$ where $C = \{(x,y,z) / x^2+y^2=1,\ z=2\}$ with the orientation of your choice by

a) directly b) using Kelvin-Stokes thm.

$\vec{F} = -3y\vec{i} + 3x\vec{j} + \vec{k}$

(11) Let R be the region in the 1^{st} octant bounded by

$$z = x, \quad z = 2 - x, \quad y = 3.$$

Find the bounds in

$$\iiint_R f \, dV = \iiint f(x,y,z)\, dx\, dy\, dz$$

$$= \iiint f(x,y,z)\, dz\, dy\, dx$$

(12) Let $D = \{(x,y) \in \mathbb{E}^2 \,/\, x^2+y^2 \leq R\}$.

- Compute $\iint_D e^{-(x^2+y^2)}\, dA_{xy}$
- What is the limit as $R \to +\infty$?
- Deduce the value of $\int_0^{+\infty} e^{-x^2}\, dx$ by a quick argument.

(13) Let $S = \{(x,y,z) \,/\, \frac{1}{4}x^2 + \frac{1}{9}y^2 = 1 \text{ \& } 0 \leq z \leq 1\}$.

- Compute Area(S). Your answer should involve some combination of $\cos^2\theta$ and $\sin^2\theta$ (elliptic integral, cannot be further simplified).
- How does Area(S) relate to Arclength(C) where

$$C = \{(x,y) \,/\, \tfrac{1}{4}x^2 + \tfrac{1}{9}y^2 = 1\} \quad ?$$