21-268 – Homework assignment week #12

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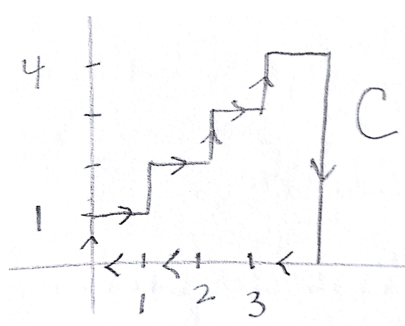
Reminder

Homework is due next Wednesdays before 5pm, to me in class or in Christopher Cox's mailbox in Wean Hall 6113 (pay attention to the arrow !). Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me your TA can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Exercises (24 pts)

1. Let $B \in \mathbb{E}$.
   (a) (2 pts) Compute
   $$\int_C 2xy^3 \mathrm{d}x + Bx^2y^2 \mathrm{d}y$$
   where C is the line segment from $(0,0)$ to $(1,1)$.
   (b) (2 pts) Same question where C is the part of the parabola $y = x^2$ that goes from $(0,0)$ to $(1,1)$.
   (c) (2 pts) Assume $B = 3$. Show that there exists $F(x,y)$ such that $\begin{bmatrix} 2xy^3 \\ 3x^2y^2 \end{bmatrix} = \vec{\nabla} F(x,y)$.
   (d) (1 pt) Explain briefly why
   $$\int_C 2xy^3 \mathrm{d}x + 3x^2y^2 \mathrm{d}y$$
   does not depend on the curve C starting from $(0,0)$ and ending at $(1,1)$.

2. (6 pts) Let $\vec{F}(x,y) = 2xy^3\vec{i} + (2x + 3x^2y^2)\vec{j}$ and let C be the curve pictured below. Find
   $$\int_C \vec{F} \cdot \mathrm{d}\vec{r}.$$



Note: direct computation of this integral would be rather long and is not the intent of the problem.

3. (6 pts) Let D be a region to which Green's theorem applies and suppose that $u(x,y)$ and $v(x,y)$ define $\mathcal{C}^2$ functions. Show that

$$\iint\limits_D \frac{\partial(u,v)}{\partial(x,y)} \mathrm{d}A_{xy} = \int_C (u\vec{\nabla}v) \cdot \mathrm{d}\vec{r}$$

where C is the curve that bounds D and has counterclockwise orientation.

4. (5 pts) Let $\vec{f} = (2x + 2xy^2 e^{x^2 y})\vec{\imath} + (3y^2 + (1 + x^2 y)e^{x^2 y})\vec{\jmath}$. Let C be any curve in the plane starting at (a,b) and ending at (c,d). Find

$$\int_C \vec{f} \cdot \mathrm{d}\vec{r}$$