21-241 – Homework assignment week #9

Laurent Dietrich
Carnegie Mellon University, Fall 2016, Sec. F and G

Reminder

Homework will be given on Fridays and due on the next Friday before 5pm, to me in class or in Andrew Zucker's mailbox in Wean Hall 6113. Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me and Andrew Zucker can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Reading

1. Poole: Sec. 4.3 and 4.4.

Exercises (19 pts)

Exercise 1 (2+4 pts)

1. Compute $\begin{bmatrix} 1/8 & 1/8 & 1/8 \\ -1/4 & 3/4 & -1/4 \\ 5/8 & -3/8 & -3/8 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ 2 & 0 & 2 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & -1 & -1 \end{bmatrix}$.
2. Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 0 & 2 \\ 3 & 3 & 1 \end{bmatrix}$.

Exercise 2 (5+1 pts)

1. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find P an invertible matrix and D a diagonal one such that

$$P^{-1}AP = D.$$

   Remark: such a P is called a *change-of-basis matrix*, and we say that A *is similar to* a diagonal matrix: A is *diagonalizable*.
2. What possible obstruction can a matrix have, that would prevent us from applying this process ? (Or to put it differently: what makes things work in the case above ?)

Exercise 3 (3+2+2 pts)

We say that two matrices $A, B \in \mathcal{M}_{nn}(\mathbb{K})$ are similar in $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ and we write $A \sim B$ if there exists an invertible matrix $P \in \mathcal{M}_{nn}(\mathbb{K})$ such that

$$P^{-1}AP = B.$$

Often, when there is no ambiguity, we will just say that A and B are similar and drop the "in $\mathbb{K}$" part.

1. Prove that similarity is an *equivalence relation*, that is for all $A, B, C \in \mathcal{M}_{nn}(\mathbb{K})$
   (a) $A \sim A$
   (b) if $A \sim B$ then $B \sim A$
   (c) if $A \sim B$ and $B \sim C$ then $A \sim C$.
2. Find all the matrices similar to I_n.
3. Show that $A = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ are not similar.