21-241 – Homework assignment week #12

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Reminder

Homework will be given on Fridays and due on the next Friday before 5pm, to me in class or in Andrew Zucker's mailbox in Wean Hall 6113. Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me and Andrew Zucker can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Reading

1. Poole: Sec. 5.2 and 5.3.

Exercises (18 pts)

Orthogonal complements (2*3 pts)

Find a basis of the orthogonal complement $W^\perp$ of W in the following cases:

$$W = \left\{ \begin{bmatrix} t/2 \\ -t/2 \\ 2t \end{bmatrix} \middle| \; t \in \mathbb{R} \right\} \qquad W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; 2x - y + 3z = 0 \right\} \qquad W = \operatorname{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\}$$

Orthogonal complement and orthogonal basis (3 pts)

Let $\{v_1, \cdots v_n\}$ be an orthogonal basis of $\mathbb{R}^n$ and let $W = \operatorname{span}\{v_1, \cdots, v_k\}$. Is it true that

$$W^\perp = \operatorname{span}\{v_{k+1}, \cdots, v_n\} \text{ ?}$$

Prove it or find a counterexample.

Some properties of orthogonal projections (3*2 pts)

Let W be a subspace of $\mathbb{R}^n$. Prove that :

1. x is in W if and only if $\operatorname{proj}_W(x) = x$.
2. x is in $W^\perp$ if and only if $\operatorname{proj}_W(x) = 0$.
3. For any $x \in \mathbb{R}^n$, $\quad \operatorname{proj}_W(\operatorname{proj}_W(x)) = \operatorname{proj}_W(x)$.

The Gram-Schmidt variations (3 pts)

The Gram-Schmidt process enables us not only to find orthogonal bases but also to complete an orthogonal set into an orthogonal basis. Indeed, one just need to combine it with the algorithm that completes an independent set of vectors into a basis that I introduced in the handout about dimension. Try it yourself by solving the following:

Find an orthogonal basis of $\mathbb{R}^3$ that contains the vector $\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$.