2. (a) $g(x) = \int_0^x f(t)\,dt$, so $g(0) = \int_0^0 f(t)\,dt = 0$.

$g(1) = \int_0^1 f(t)\,dt = \frac{1}{2} \cdot 1 \cdot 1$ [area of triangle] $= \frac{1}{2}$.

$g(2) = \int_0^2 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^2 f(t)\,dt$ [below the x-axis]

$= \frac{1}{2} - \frac{1}{2} \cdot 1 \cdot 1 = 0.$

$g(3) = g(2) + \int_2^3 f(t)\,dt = 0 - \frac{1}{2} \cdot 1 \cdot 1 = -\frac{1}{2}$.

$g(4) = g(3) + \int_3^4 f(t)\,dt = -\frac{1}{2} + \frac{1}{2} \cdot 1 \cdot 1 = 0$.

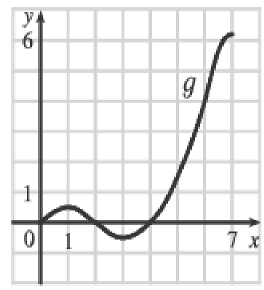
$g(5) = g(4) + \int_4^5 f(t)\,dt = 0 + 1.5 = 1.5$.

$g(6) = g(5) + \int_5^6 f(t)\,dt = 1.5 + 2.5 = 4$.

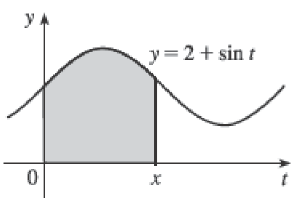
(b) $g(7) = g(6) + \int_6^7 f(t)\,dt \approx 4 + 2.2$ [estimate from the graph] $= 6.2$.

(c) The answers from part (a) and part (b) indicate that g has a minimum at $x = 3$ and a maximum at $x = 7$. This makes sense from the graph of f since we are subtracting area on $1 < x < 3$ and adding area on $3 < x < 7$.

(d)



6.



(a) By FTC1 with $f(t) = 2 + \sin t$ and $a = 0$, $g(x) = \int_0^x (2 + \sin t)\,dt \Rightarrow$

$g'(x) = f(x) = 2 + \sin x$.

(b) Using FTC2,

$g(x) = \int_0^x (2 + \sin t)\,dt = [2t - \cos t]_0^x = (2x - \cos x) - (0 - 1)$

$= 2x - \cos x + 1 \Rightarrow$

$g'(x) = 2 - (-\sin x) + 0 = 2 + \sin x$

12. $G(x) = \displaystyle\int_x^1 \cos\sqrt{t}\,dt = -\int_1^x \cos\sqrt{t}\,dt \Rightarrow G'(x) = -\frac{d}{dx}\int_1^x \cos\sqrt{t}\,dt = -\cos\sqrt{x}$

18. Let $u = \sin x$. Then $\dfrac{du}{dx} = \cos x$. Also, $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$, so

$$y' = \frac{d}{dx}\int_{\sin x}^1 \sqrt{1+t^2}\,dt = \frac{d}{du}\int_u^1 \sqrt{1+t^2}\,dt \cdot \frac{du}{dx} = -\frac{d}{du}\int_1^u \sqrt{1+t^2}\,dt \cdot \frac{du}{dx}$$
$$= -\sqrt{1+u^2}\,\cos x = -\sqrt{1+\sin^2 x}\,\cos x$$

24. $\displaystyle\int_1^8 x^{-2/3}\,dx = \left[\frac{x^{1/3}}{1/3}\right]_1^8 = 3\left[x^{1/3}\right]_1^8 = 3(8^{1/3} - 1^{1/3}) = 3(2-1) = 3$

30. $\int_0^2 (y-1)(2y+1)\,dy = \int_0^2 (2y^2 - y - 1)\,dy = \left[\frac{2}{3}y^3 - \frac{1}{2}y^2 - y\right]_0^2 = \left(\frac{16}{3} - 2 - 2\right) - 0 = \frac{4}{3}$

38. $\int_0^1 \cosh t\,dt = \left[\sinh t\right]_0^1 = \sinh 1 - \sinh 0 = \sinh 1 \quad \left[\text{or } \frac{1}{2}(e - e^{-1})\right]$

40. $\displaystyle\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 (4u^{-3} + u^{-1})\,du = \left[\frac{4}{-2}u^{-2} + \ln|u|\right]_1^2 = \left[\frac{-2}{u^2} + \ln u\right]_1^2 = \left(-\tfrac{1}{2} + \ln 2\right) - (-2 + \ln 1) = \tfrac{3}{2} + \ln 2$

56. $\displaystyle g(x) = \int_{1-2x}^{1+2x} t\sin t\,dt = \int_{1-2x}^{0} t\sin t\,dt + \int_0^{1+2x} t\sin t\,dt = -\int_0^{1-2x} t\sin t\,dt + \int_0^{1+2x} t\sin t\,dt \quad \Rightarrow$

$$g'(x) = -(1-2x)\sin(1-2x)\cdot\frac{d}{dx}(1-2x) + (1+2x)\sin(1+2x)\cdot\frac{d}{dx}(1+2x)$$
$$= 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x)$$

60. $\displaystyle f(x) = \int_0^x (1-t^2)e^{t^2}\,dt$ is increasing when $f'(x) = (1-x^2)e^{x^2}$ is positive.

Since $e^{x^2} > 0$, $f'(x) > 0 \quad\Leftrightarrow\quad 1 - x^2 > 0 \quad\Leftrightarrow\quad |x| < 1$, so f is increasing on $(-1, 1)$.

2. $\displaystyle\frac{d}{dx}\left(\frac{1}{2}x + \frac{1}{4}\sin 2x + C\right) = \frac{1}{2} + \frac{1}{4}\cos 2x \cdot 2 + 0 = \frac{1}{2} + \frac{1}{2}\cos 2x$

$$= \tfrac{1}{2} + \tfrac{1}{2}\left(2\cos^2 x - 1\right) = \tfrac{1}{2} + \cos^2 x - \tfrac{1}{2} = \cos^2 x$$

6. $\displaystyle\int\left(\sqrt{x^3} + \sqrt[3]{x^2}\right)dx = \int (x^{3/2} + x^{2/3})\,dx = \frac{x^{5/2}}{5/2} + \frac{x^{5/3}}{5/3} + C = \tfrac{2}{5}x^{5/2} + \tfrac{3}{5}x^{5/3} + C$

12. $\displaystyle\int\left(x^2 + 1 + \frac{1}{x^2+1}\right)dx = \frac{x^3}{3} + x + \tan^{-1}x + C$

18. $\displaystyle\int \frac{\sin 2x}{\sin x}\,dx = \int \frac{2\sin x\,\cos x}{\sin x}\,dx = \int 2\cos x\,dx = 2\sin x + C$

26. $\int_{-1}^1 t(1-t)^2\,dt = \int_{-1}^1 t(1-2t+t^2)\,dt = \int_{-1}^1 (t - 2t^2 + t^3)\,dt = \left[\frac{1}{2}t^2 - \frac{2}{3}t^3 + \frac{1}{4}t^4\right]_{-1}^1$

$$= \left(\tfrac{1}{2} - \tfrac{2}{3} + \tfrac{1}{4}\right) - \left(\tfrac{1}{2} + \tfrac{2}{3} + \tfrac{1}{4}\right) = -\tfrac{4}{3}$$

32. $\displaystyle\int_1^4 \frac{\sqrt{y} - y}{y^2}\,dy = \int_1^4\left(\frac{\sqrt{y}}{y^2} - \frac{y}{y^2}\right)dy = \int_1^4 (y^{-3/2} - y^{-1})\,dy = \left[-2y^{-1/2} - \ln|y|\right]_1^4 = \left[-\frac{2}{\sqrt{y}} - \ln|y|\right]_1^4$

$$= (-1 - \ln 4) - (-2 - \ln 1) = 1 - \ln 4$$

42. $\displaystyle\int_1^2 \frac{(x-1)^3}{x^2}\,dx = \int_1^2 \frac{x^3 - 3x^2 + 3x - 1}{x^2}\,dx = \int_1^2\left(x - 3 + \frac{3}{x} - \frac{1}{x^2}\right)dx = \left[\tfrac{1}{2}x^2 - 3x + 3\ln|x| + \frac{1}{x}\right]_1^2$

$$= \left(2 - 6 + 3\ln 2 + \tfrac{1}{2}\right) - \left(\tfrac{1}{2} - 3 + 0 + 1\right) = 3\ln 2 - 2$$

50. $y = \sqrt[4]{x} \quad\Rightarrow\quad x = y^4$, so $A = \int_0^1 y^4\,dy = \left[\frac{1}{5}y^5\right]_0^1 = \frac{1}{5}$.

56. The slope of the trail is the rate of change of the elevation E, so $f(x) = E'(x)$. By the Net Change Theorem, $\int_3^5 f(x)\,dx = \int_3^5 E'(x)\,dx = E(5) - E(3)$ is the change in the elevation E between $x = 3$ miles and $x = 5$ miles from the start of the trail.

64. By the Net Change Theorem, the amount of water that flows from the tank during the first 10 minutes is

$\int_0^{10} r(t)\,dt = \int_0^{10}(200 - 4t)\,dt = \left[200t - 2t^2\right]_0^{10} = (2000 - 200) - 0 = 1800$ liters.

66. (a) By the Net Change Theorem, the total amount spewed into the atmosphere is $Q(6) - Q(0) = \int_0^6 r(t)\,dt = Q(6)$ since $Q(0) = 0$. The rate $r(t)$ is positive, so Q is an increasing function. Thus, an upper estimate for $Q(6)$ is R_6 and a lower estimate for $Q(6)$ is L_6. $\Delta t = \dfrac{b-a}{n} = \dfrac{6-0}{6} = 1$.

$R_6 = \sum_{i=1}^{6} r(t_i)\,\Delta t = 10 + 24 + 36 + 46 + 54 + 60 = 230$ tonnes.

$L_6 = \sum_{i=1}^{6} r(t_{i-1})\,\Delta t = R_6 + r(0) - r(6) = 230 + 2 - 60 = 172$ tonnes.

(b) $\Delta t = \dfrac{b-a}{n} = \dfrac{6-0}{3} = 2$. $Q(6) \approx M_3 = 2[r(1) + r(3) + r(5)] = 2(10 + 36 + 54) = 2(100) = 200$ tonnes.

70. Let $M(t)$ denote the number of megabits transmitted at time t (in hours) [note that $D(t)$ is measured in megabits/second]. By the Net Change Theorem and the Midpoint Rule,

$$M(8) - M(0) = \int_0^8 3600D(t)\,dt \approx 3600 \cdot \tfrac{8-0}{4}[D(1) + D(3) + D(5) + D(7)]$$

$$\approx 7200(0.32 + 0.50 + 0.56 + 0.83) = 7200(2.21) = 15{,}912 \text{ megabits}$$