Concepts Counting Two Ways

1. Prove the following by counting 2 ways. This is an arithmetic sum formula.

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

2. Prove the following by counting 2 ways when q is an integer greater than 1. This is a geometric sum formula.

$$\sum_{i=0}^{n-1} q^i = \frac{q^n - 1}{q - 1}$$

3. Prove the following by counting 2 ways.

$$\sum_{k=0}^{n} \binom{x+k}{k} = \binom{x+n+1}{n}$$