21-320 Homework Set 8

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Due Fri 5 May. I encourage submission by email (preferably as a PostScript file) to `maplehw@asdb.math.cmu.edu`.

1. Let g be the function such that

   (a) g is periodic of period 2π.

   (b) $g(t) = t + \pi$ for $-\pi \leq t \leq -\pi/2$.

   (c) $g(t) = -t$ for $-\pi/2 \leq t \leq 0$.

   (d) $g(t) = 0$ for $0 \leq t \leq \pi$.

   Find the Fourier series for g.

   Write a Maple function that takes arguments N and x and returns

   $$f(N, x) = c_0/2 + \sum_{j=0}^{N} c_j \cos(jx) + \sum_{j=0}^{N} d_j \sin(jx),$$

   where $c_0/2 + \sum_{j=0}^{\infty} c_j \cos(jx) + \sum_{j=0}^{\infty} d_j \sin(jx)$ is the Fourier series for g.

   Draw the graphs of $f(10, x)$ and $f(100, x)$ in the interval $[-\pi, +\pi]$.

2. Use Maple to show that any function $f(x, t)$ of the form $G(x + t) + H(x - t)$ solves the wave equation $f_{xx} = f_{tt}$, assuming that G and H are infinitely differentiable.

   Now let $G(y) = e^{-y^2}$ and $H(y) = -e^{-y^2}$. Make an animation showing the behaviour of $f(x, t)$ in the range $x \in [-10, +10]$ as time t goes from -5 to $+5$.

3. The behaviour of a square elastic membrane clamped at the edges can be modelled as follows: let $f(x, y, t)$ be the displacement of (x, y) in the z-direction at time t. Then we have a partial differential equation $f_{xx} + f_{yy} = f_{tt}$ and the boundary coditions $f(-\pi, y, t) = f(\pi, y, t) = f(x, -\pi, t) = f(x, \pi, t) = 0$.

   Find a non-zero function f that satisfies this equation and these boundary conditions, and make an animation showing how it behaves.

   Hint: look for solutions in the form $X(x)Y(y)T(t)$.