21-320 Homework Set 6

James Cummings

March 14, 2000

Due Mon 20 March. I encourage submission by email (preferably as a PostScript file) to `maplehw@asdb.math.cmu.edu`.

1. Read the help page on the function rand. Write a function f which takes a non-negative integer n as argument, generates 10^n uniformly distributed random points in the square given by $-1 \leq y \leq 1$, $-1 \leq x \leq 1$, and returns the value $M/10^n$ where M is the number of the 10^n points such that $x^2 + y^2 \leq 1$. Compute $f(i)$ for i from 0 to 6 (or more if you have fast hardware). How would you expect f to behave for large n?

2. Write a function $f0$ which takes a non-negative integer n as argument, computes $\texttt{randmatrix}(\mathtt{n}, \mathtt{n})$ and then computes how much time it takes to invert this matrix. Write another function $f1$ which takes a non-negative integer n as argument, computes $\texttt{randmatrix}(\mathtt{n}, \mathtt{n})$, converts the matrix to floating point and then computes how much time it takes to invert this matrix.

   Compute as many values of $f0$ and $f1$ as you can. Plot graphs of $ln(f0(n))$ against n, $ln(f0(n))$ against $ln(n)$, $ln(f1(n))$ against n, and $ln(f1(n))$ against $ln(n)$.

   Roughly how large would you expect $f0(100)$ and $f1(100)$ to be?

3. Let F be the vector field given by $F(x, y, z) = (yz, zx, xy)$. Let C be the curve parametrised by $(t\cos(t), t\sin(t), t)$ as t ranges from 0 to 2π. Draw a picture of C. Compute the integral of F along C.

4. Generate a 5×5 Hilbert matrix. Generate a random vector b of height 5 using randvector. Solve the equation $Ax = b$ exactly using linsolve,

to get a vector $x0$. Now convert A to floating point and solve $Ax = b$ numerically, getting a vector $x1$.

Compute $x1 - x0$, $Ax_0 - b$, $Ax_1 - b$. Do these results suggest that a vector which approximately solves $Ax = b$ should be a reasonable approximation to an exact solution?