21-320 Homework Set 4

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Due Mon 21 February. I encourage submission by email (preferably as a PostScript file) to `maplehw@asdb.math.cmu.edu`. Please make sure you put your name at the top of the worksheet, and clearly indicate which calculations are related to which questions.

1. Using the "inert form" Int of the integral operator, write down a Maple expression corresponding to the multiple integral

$$\int_0^1 \int_0^a \int_0^{\frac{a-b}{2}} \int_0^1 \int_{\frac{d(b+c)}{a-c}}^1 \int_0^{\frac{ae-bd-cd-ce}{a+b}} \int_0^1 \int_0^1 \int_0^{\frac{x(bf+ce)+y(af+cd)}{ae-bd}} 1 \, dz \, dy \, dx \, df \, de \, dd \, dc \, db \, da.$$

   Attempt to evaluate the integral numerically (apply evalf to the expression written with Int). Now replace all the Int's by int's and attempt to evaluate the integral symbolically. Comment on your results.

2. For each $k \geq 1$ let $p_k(n)$ be the polynomial in n given by

$$p_k(n) = \frac{n(n+1)\ldots(n+k-1)}{k!}.$$

   Use Maple to prove that $\sum_{i=1}^{n} p_3(n) = p_4(n)$.

3. For $k \geq 1$ let f_k be the function given by

$$f_n(x) = \pi x \prod_{k=1}^{n} (1 - x^2/k^2).$$

   Plot the graphs of f_{10}, f_{100} and f_{1000} for x in $[0, 2\pi]$ and $y \in [-1, +1]$. How would you conjecture that f_i behaves as i becomes larger?

4. Compute the partial derivatives of $\sec(x+y)$ with respect to x and y using

   (a) The diff operator.

   (b) The D operator.

5. Let $s_n = \sum_{j=1}^{n} \cos(1/j)$. Compute (numerically) s_{10^k} for $k \leq 6$. Would you guess that the infinite series $s = \sum_{j=1}^{\infty} \cos(1/j)$ converges?

6. Let $f(x)$ be given by $f(x) = e^{-x^{-2}}$ for $x \neq 0$ and $f(0) = 0$. Plot the graph of f in the interval $[-1, +1]$. Find the first, second and third derivatives of f at the point $x = 0$.