21-320 Homework Set 3

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Due Mon 14 February. I encourage submission by email (preferably as a PostScript file) to `maplehw@asdb.math.cmu.edu`.

1. Let the cubic polynomial $x^3 + ax^2 + bx + c$ have roots r_1, r_2, r_3. Find expressions for $r_1^n + r_2^n + r_3^n$ as polynomials in a, b, c for $n = 0, 1, 2, 3$
2. Plot the graph of the equation $y^2 = x * (x - 1) * (x + 1)$ in the region where x and y both lie between -3 and 3. Find the point on the graph where $x = -0.5$ and y is positive. Find the tangent line at that point, and find all other points at which the tangent line meets the graph. You should give your answers to an accuracy of 5 decimal places.
3. Find all the (complex) roots of $x^5 = x + 1$ to an accuracy of 20 decimal places. Compute the sum of these roots. Explain your answer.
4. Plot the graphs of the real and imaginary parts of $\ln(z)$ where $z = x + y * i$ is a complex number and both x, y lie in $[-1, 1]$. Find complex numbers z and w where $|z - w| < 10^{-5}$ and $|\ln(z) - \ln(w)| > 1$.
5. Let r_n be the nth positive solution of $x = \tan(x)$. Compute r_n and $\frac{r_n}{\pi/2+(n+1)\pi}$ for $1 \leq n \leq 10$. Explain your results.