**21-300 F15 HW 4**

Since it's harder to word-process proofs, you may submit hand-written solutions to this homework subject to the following conditions: your homework is written in black ink on alternating lines, and is submitted as a legible 600dpi scan in PDF form.

Note: When I say "prove" in this homework I mean produce a formal proof using the proof rules from class. Unless I specify hypotheses, your proof should have no hypotheses. Letters from the Greek alphabet ($\alpha$, $\beta$, $\gamma$, $\delta$) stand for arbitrary formulae. Your proofs should be labelled so that it is clear which proof rule was used at which stage.

IMPORTANT: PLEASE EMAIL COMPLETED HOMEWORKS TO:

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(1) Prove by induction on the term $\rho$ that if $x$ and $y$ are distinct variables and $\sigma$ and $\tau$ are closed terms, then $\rho[x/\sigma][y/\tau] = \rho[y/\tau][x/\sigma]$.

(2) Prove the formulae:
   (a) $\exists x\ \phi \to \neg\forall x\ \neg\phi$.
   (b) $\neg\forall x\ \neg\phi \to \exists x\ \phi$.
   (c) $\forall x\ \phi \to \neg\exists x\ \neg\phi$.
   (d) $\neg\exists x\ \neg\phi \to \forall x\ \phi$.
   (e) $\exists x\ (\phi \vee \psi) \to (\exists x\ \phi \vee \exists x\ \psi)$
   (f) $(\exists x\ \phi \vee \exists x\ \psi) \to \exists x\ (\phi \vee \psi)$

(3) Let $\phi$ be a formula in some first order language, and suppose that $\phi$ has exactly two free variables $x$ and $y$. Let $\mathcal{M}$ be a structure for the language. Prove that $\mathcal{M} \models \forall x\ \forall y\ \phi$ if and only if $\mathcal{M} \models \forall y\ \forall x\ \phi$.