21-300 F15 HW 3

IMPORTANT: PLEASE EMAIL COMPLETED HOMEWORKS TO:

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(1) Recall that the *deductive closure* of a set Γ of propositional formulae is $\{\delta : \Gamma \vdash \delta\}$. A set is *deductively closed* if and only if it is equal to its own deductive closure. Use the Soundness and Completeness theorems to give a short proof that for any Γ the deductive closure of Γ is deductively closed (if your proof uses induction or is more than ten lines long then it is probably too long)

(2) Prove that no finite consistent set of propositional formulae is complete.

(3) Let Γ be a set of propositional formulae. Prove that if every finite subset of Γ is consistent, then Γ is consistent.

(4) Two formulae of propositional logic are *equivalent* if they are satisfied by exactly the same truth assignments.

   (a) Prove that every propositional formula is equivalent to one which only uses $\neg$ and $\wedge$.

   (b) Prove that not every propositional formula is equivalent to one which only uses $\rightarrow$.

(5) Consider a signature for a first order language which has only three symbols, a binary function symbol $\circ$, a constant symbol e, and a binary relation symbol $\equiv$ (whose intended interpretation is the equlaity relation). Write down sentences which express:

   (a) $\circ$ is commutative.

   (b) $\circ$ is associative.

   (c) e is a two-sided identity element for $\circ$.