21-300 F15 HW 2

Since it's harder to word-process proofs, you may submit hand-written solutions to this homework subject to the following conditions: your homework is written in black ink on alternating lines, and is submitted as a legible 600dpi scan in PDF form.

Note: When I say "prove" in this homework I mean produce a formal proof using the nine proof rules from class. Unless I specify hypotheses, your proof should have no hypotheses. Letters from the Greek alphabet (α, β, γ, δ) stand for arbitrary formulae.

IMPORTANT: PLEASE EMAIL COMPLETED HOMEWORKS TO:

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(1) Prove the formulae:
   (a) $(\alpha \to \beta) \to ((\neg\alpha) \vee \beta)$.
   (b) $((\neg\alpha) \vee \beta) \to (\alpha \to \beta)$.
   (c) $(\alpha \to \beta) \to ((\neg\beta) \to (\neg\alpha))$.
   (d) $((\neg\beta) \to (\neg\alpha)) \to (\alpha \to \beta)$
(2) Prove the following formulae without using the Contradiction rule (that is the last rule from class, that lets you take a proof with conclusion $\neg\neg\beta$ and make a proof with the same hypotheses and conclusion β).
   (a) $\neg\beta \to \neg\neg\neg\beta$.
   (b) $\neg\neg\neg\beta \to \neg\beta$.
(3) Prove the formula $\beta \vee \gamma$ from the hypotheses $\alpha \vee \beta$, $\neg\alpha \vee \gamma$.
(4) A consistent theory Γ is said to be *maximally consistent* if there no consistent Δ with $\Gamma \subsetneq \Delta$. Prove that a maximally consistent theory is deductively closed. Hint: Suppose it isn't, what would be a natural thing to try adding?