DISCRETE MATHEMATICS 21228 — HOMEWORK 9

JC

Due in class Wednesday November 19. You may collaborate but *must* write up your solutions by yourself.

Late homework will not be accepted. Homework must either be typed or written legibly in blue or black ink on alternate lines, illegible homework will be returned ungraded (so you can rewrite it legibly).

Please write the name of your recitation instructor and the time and place of your recitation at the top of your homework.

IMPORTANT: We are now following the convention that all graphs are finite unless explicitly stated to be infinite.

(1) Let a and b be distinct vertices in a connected graph G. An *Eulerian trail* from a to b is a trail (a walk without repeated edges) from a to b which visits each edge exactly once. Prove that such a trail exists if and only if a and b are the only vertices of odd degree in G.

(2) For a graph H let $p_H(n)$ be the number of vertex colourings of H using colours $1, ...n$. Find a formula for $p_H(n)$ for the following graphs H:
  (a) The complete graph K_m (the graph with m vertices where every pair is joined by an edge).
  (b) The m-cycle C_m.
  (c) The graph obtained from a 6-cycle by joining two opposite vertices.
  (d) The tree with vertices $\{a, b, c, d, e\}$ and edges $\{ab, ac, bd, be\}$.

(3) Prove that a graph is planar if and only if it can be drawn on the surface of a sphere.

(4) (Tricky) Find a graph which can be drawn on the surface of a torus with chromatic number greater than four. Extra credit if you can achieve the optimal value of seven!

(5) Show that a graph G has at least $\binom{\chi(G)}{2}$ edges.