DISCRETE MATHEMATICS 21228 — HOMEWORK 7

JC

Due in class Wednesday October 29. You may collaborate but *must* write up your solutions by yourself.

Late homework will not be accepted. Homework must either be typed or written legibly in blue or black ink on alternate lines, illegible homework will be returned ungraded (so you can rewrite it legibly).

Please write the name of your recitation instructor and the time and place of your recitation at the top of your homework.

(1) A *clique* in a graph is a set of vertices every pair of which is joined by an edge. An *independent set* is a set of vertices no pair of which is joined by an edge. Prove that every graph with an infinite set of vertices contains either an infinite clique or an infinite independent set.

(2) Consider the following graph. The vertex set is

$$V = \{1/2^n : n \in \mathbb{N}\} \cup \{-1/2^n : n \in \mathbb{N}\}.$$

The edge set is

$$E = \{\{1/2^n, 1/2^{n+1}\} : n \in \mathbb{N}\} \cup \{\{-1/2^n, -1/2^{n+1}\} : n \in \mathbb{N}\}.$$

How many connected components are there?

(3) (Rather an open-ended one) In class we saw that a triangle-free graph on n vertices has at most $n^2/4$ vertices. Describe a construction that gives for each n a triangle-free graph on n vertices with as many edges as you can manage. You will get full credit for any construction where the number of edges grows as a quadratic polynomial function of n.

(4) Given n vertices, we create a "random graph" by tossing a fair coin $\binom{n}{2}$ times to decide whether each pair of vertices should be joined by an edge. Estimate the probability that the resulting graph is triangle free. What is the expected number of edges?

(5) A graph is a *tree* if it is connected and has no cycles. Find the number of trees on n vertices for $n \leq 5$ (here you should consider two trees to be the same if they are "isomorphic", that is to say there is a bijection between their vertex sets which induces a

bijection between their edge sets; to put it more intutively the graphs have the same structure, as an example any two k-cycles are isomorphic). Conjecture a formula for the number of trees on n vertices.