DISCRETE MATHEMATICS 21228 — HOMEWORK 5

JC

Due in class Wednesday October 8. You may collaborate but *must* write up your solutions by yourself.

Late homework will not be accepted. Homework must either be typed or written legibly in blue or black ink on alternate lines, illegible homework will be returned ungraded (so you can rewrite it legibly).

Please write the name of your recitation instructor and the time and place of your recitation at the top of your homework.

(1) $R(a, b, c)$ is (by definition) the least N such that every colouring of the pairs from an N-element set in three colours (say red, green and blue) has a homogeneous red set of size a, a homogeneous blue set of size b, or a homogeneous green set of size c.
   (a) Find $R(2, 2, n)$.
   (b) Prove that for $a, b, c > 2$ we have $R(a, b, c) \leq R(a-1, b, c) + R(a, b-1, c) + R(a, b, c-1)$.

(2) Find an upper bound for $R(n, n, n)$.

(3) Use the probabilistic method to find a lower bound for $R(n, n, n)$.

(4) Let F be the set of all finite subsets of $\mathbb{N}$. Is the following Ramsey-type statement true or false?
   "For all $f : F \longrightarrow \{\text{red}, \text{blue}\}$ there is an infinite set $H \subseteq \mathbb{N}$ such that all the finite subsets of H are given the same colour by f".

(5) Prove that if $R(s-1, t)$ and $R(s, t-1)$ are both even then $R(s, t) \leq R(s-1, t) + R(s, t-1) - 1$.

(6) (Tricky!) Prove that $R(3, 4) = 9$.