COMMUTATIVE ALGEBRA HW 9

JC

Due in class Mon 26 September.

(1) Prove that for every ring R there is a unique ring HM from $\mathbb{Z}$ to R.

(2) By Q1 every ring has a unique $\mathbb{Z}$-algebra structure (in rather the same way that every abelian group has a unique $\mathbb{Z}$-module structure, so that we blur the distinction).

Let R and S be rings. Show that $R \otimes_{\mathbb{Z}} S$ can be made into a ring in such a way that $(r_1 \otimes s_1) \times (r_2 \otimes s_2) = r_1 r_2 \otimes s_1 s_2$. You should verify the ring axioms! (Look on p30 of A and M if you get stuck).

(3) Verify that the ring $R \otimes S$ together with the maps $r \mapsto r \otimes 1$ and $s \mapsto 1 \otimes s$ constitute a coproduct in the category of rings. Hint: given $\phi : R \to T$ and $\psi : S \to T$ how can we cook up a $\mathbb{Z}$-bilinear map from $R \times S$ to T?