COMMUTATIVE ALGEBRA HW 7

JC

Due in class Wed 21 September.

(1) (A and M 1.19) A topological space is said to be *irreducible* iff it is nonempty and every pair of nonempty open sets has a nonempty intersection. Let R be a ring. Show that $Spec(R)$ is irreducible iff the nilradical is prime.

(2) (A and M 3.1 with a twist) Let M be an fg R-module and let $S \subseteq R$ be a MC set. Show that $S^{-1}M = \{0\}$ iff there is $s \in S$ such that $sM = \{0\}$. What if M is not fg?

(3) Let R be a ring. Show that every minimal prime ideal of R is contained in the set of zero-divisors of R.