COMMUTATIVE ALGEBRA HW 2

JC

Due in class Mon 5 September.

(1) A prime ideal J is *minimal* iff for all prime $I \subseteq J$ we have $I = J$. Show that every prime ideal contains a minimal prime ideal.
(2) Identify the radical ideals of $\mathbb{Z}$.
(3) Let R be a ring. The *power series ring* $R[[x]]$ consists of all expressions $\sum_{i=0}^{\infty} r_i x^i$ with the obvious definitions of $+$ and $\times$. Identify the units in $R[[x]]$.
(4) Let $\phi : R \to S$ be a ring HM and let J be an ideal of S. Show that if $I = \phi^{-1}[J]$ then I is an ideal of R, R/I is isomorphic to a subring of S/J, and "J is prime" implies "I is prime".