

## COMMUTATIVE ALGEBRA HW 2

JC

Due in class Mon 5 September.

- (1) A prime ideal  $J$  is *minimal* iff for all prime  $I \subseteq J$  we have  $I = J$ . Show that every prime ideal contains a minimal prime ideal.
- (2) Identify the radical ideals of  $\mathbb{Z}$ .
- (3) Let  $R$  be a ring. The *power series ring*  $R[[x]]$  consists of all expressions  $\sum_{i=0}^{\infty} r_i x^i$  with the obvious definitions of  $+$  and  $\times$ . Identify the units in  $R[[x]]$ .
- (4) Let  $\phi : R \rightarrow S$  be a ring HM and let  $J$  be an ideal of  $S$ . Show that if  $I = \phi^{-1}[J]$  then  $I$  is an ideal of  $R$ ,  $R/I$  is isomorphic to a subring of  $S/J$ , and “ $J$  is prime” implies “ $I$  is prime”.