COMMUTATIVE ALGEBRA HW 15

JC

Due in class Wed 19 October.

(1) A topological space is called *discrete* if all subsets are open. Show that if R is Artinian then $Spec(R)$ is discrete.

(2) (A and M 6.4) Let M be a Noetherian R-module and let I be the *annihilator* of M, that is $\{a : \forall m \; am = 0\}$.

Show that R/I is a Noetherian ring. Show by example that R need not be Noetherian.

(3) Let R be a local Noetherian integral domain of dimension one.

Show that R has a unique nonzero prime ideal P, and that every nonzero ideal of R is P-primary.