## COMMUTATIVE ALGEBRA HW 15

## $\mathrm{JC}$

Due in class Wed 19 October.

- (1) A topological space is called *discrete* if all subsets are open. Show that if R is Artinian then Spec(R) is discrete.
- (2) (A and M 6.4) Let M be a Noetherian R-module and let I be the *annihilator* of M, that is  $\{a : \forall m \ am = 0\}$ . Show that R/I is a Noetherian ring. Show by example that R need not be Noetherian.
- (3) Let R be a local Noetherian integral domain of dimension one. Show that R has a unique nonzero prime ideal P, and that every nonzero ideal of R is P-primary.