COMMUTATIVE ALGEBRA HW 13

JC

Due in class Fri 7 October.

(1) Let B be a ring. Show there exists a ring C with $B \leq C$ and an element $c \in C \setminus B$ such that c is integral over B.
(2) Let R be a ring, I an ideal of R and M an R-module. Show that M/IM and $R/I \otimes_R M$ are isomorphic as R/I-modules.
(3) Let A, B be rings with $A \leq B$ and suppose that $B \setminus A$ is closed under multiplication. Show that the integral closure of A in B (the set of elements of B which are integral over A) is precisely A.