COMMUTATIVE ALGEBRA HW 11

JC

Due in class Mon 3 October.

(1) Let M be an R-module and define an $R[x]$-module $M[x]$ as follows: the elements are all finite sums $\sum_{i=0}^{n} m_i x^i$ where $n \in \mathbb{N}$ and $m_i \in M$, with the obvious operations for $+$ and scalar multiplication. Show that $M[x]$ is isomorphic to $R[x] \otimes_R M$ as an $R[x]$-module. Hint: what is the obvious R-bilinear map from $R[x] \times M$ to $M[x]$?

(2) Show that if M is a Noetherian R-module then $M[x]$ is a Noetherian $R[x]$-module. Hint: what is this saying when $M = R$?

(3) Let I be an ideal of R and let $S = 1 + I$, that is to say $\{1 + a : a \in I\}$. Show that
   (a) S is a multiplicatively closed subset of R.
   (b) Let $S^{-1}I$ be the extension of I in $S^{-1}R$, that is to say $S^{-1}I = \{a/s : a \in I, s \in S\}$. Show that $S^{-1}I$ is contained in the Jacobson radical of $S^{-1}R$.