COMMUTATIVE ALGEBRA HW 10

JC

Due in class Fri 30 September.

(1) Read the handout on affine algebraic geometry.

(2) Let $k = \mathbb{C}$. Consider the variety $V(I)$ in $\mathbb{A}^3$ where $I = (x^2 - yz, xz - x)$. Show that V is the irredundant union of three irreducible varieties, and describe them by giving the prime ideal corresponding to each one.

(3) (A small part of A and M 3.21)

A *homeomorphism* between two topological spaces X and Y is a bijection f between X and Y such that f and f^{-1} are both continuous (so f sets up a bijection between the open sets of X and of Y via the correspondence $O \mapsto f[O]$).

Let R be a ring, S an MC subset of R and $\phi : R \to S^{-1}R$ the map $\phi : r \mapsto r/1$. Let $X = Spec(R)$ and $Y = Spec(S^{-1}R)$ so that as we saw in a previous HW ϕ induces a continuous map $Spec(\phi)$ from Y to X. Let Z be the image of $Spec(\phi)$, and give Z the subspace topology. Show that $Spec(\phi)$ is a homeomorphism from Y to Z.