ALGEBRA HOMEWORK SET I

JAMES CUMMINGS

You may collaborate on this homework set, but must write up your solutions by yourself. Please contact me by email if you are puzzled by something, would like a hint or believe that you have found a typo.

(1) Let p be prime and let G be a non-abelian group of order p^3. Show that $Z(G) = [G, G]$ and this is a subgroup of order p.
(2) Prove that every group of order 15 is cyclic
(3) Prove there is no simple group of order pq for distinct primes p, q.
(4) Repeat the previous exercise for groups of order p^2q.
(5) Let G be a simple group of order 60. Determine the numbers of Sylow p-subgroups for $p = 2, 3, 5$ (if you can't determine the exact values give as much information as you can).
(6) Let F be a finite field with q elements and let $GL_n(F)$ be the group of invertible linear maps from F^n to F^n. What is the order of G?
(7) Prove or give a counterxample to the statement "if G is nilpotent, every subgroup of G is also nilpotent".
(8) Prove that there are (up to isomorphism) only two non-abelian groups of order 8 and describe them.
(9) Recall that if G is a group then $G^{(n)}$ is defined by the induction $G^{(0)} = G$, $G^{(n+1)} = [G^{(n)}, G^{(n)}]$. Compute this series of groups when $G = S_3$, S_4, S_5.