ALGEBRA HOMEWORK SET II

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You may collaborate on this homework set, but must write up your solutions by yourself. Please contact me by email if you are puzzled by something, would like a hint or believe that you have found a typo.

NOTE: This is one is shorter because it's late. Apologies for the delay.

C_n denotes a cyclic group of order n.

(1) Use Sylow's theorem to show that there is no simple group of any of the following orders: 20, 26, 34, 48.

(2) Describe the automorphism group $Aut(C_5)$. Find all the HMs from G to $Aut(C_5)$ for
   (a) $G = C_4$.
   (b) $G = C_2 \times C_2$.

(3) Suppose that $G = HN$ where $H \leq G$, $N \lhd G$ and $H \cap N = \{e\}$.
   (a) Show that every element of G can be written as hn for unique elements $h \in H$ and $n \in N$.
   (b) Express $h_1 n_1 h_2 n_2$ where $h_i \in H$, $n_i \in N$ in the form hn with $h \in H$, $n \in N$.

(4) Suppose that H, N are groups and $\psi : H \to Aut(N)$ is a HM. Define a binary operation $*$ on the set of ordered pairs $H \times N$ as follows:

$$(h_1, n_1) * (h_2, n_2) = (h_1 \times_H h_2, \psi(h_2^{-1})(n_1) \times_N n_2).$$

   Prove that this binary operation makes $H \times N$ into a group.

(5) Explain the relationship between the previous two questions. Use Sylow's theorem and the ideas of the previous two questions to classify groups of order 20 up to isomorphism.

   Hint: there may be unexpected IMs among the groups that you construct, be careful.