Algebra Homework Set 6

James Cummings (jcumming@andrew.cmu.edu)

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1. Let F be the subfield of $\mathbb{C}$ generated over $\mathbb{Q}$ by the roots of $x^4 - 2$. Show that F is a Galois extension of $\mathbb{Q}$ and that the Galois group is dihedral of order 8. Find all the subgroups of the Galois group and all the intermediate fields. Which intermediate fields are Galois extensions of $\mathbb{Q}$?

   (You may want to remind yourself about Eisenstein's criterion)

2. Let E be a field. As usual $E[x]$ is the ring of polynomials and $E(x)$ is the field of fractions of $E[x]$. Show that every element σ of $G = \Gamma(E(x)/E)$ is determined by $\sigma(x)$, and that the possible values of $\sigma(x)$ are exactly the elements of form
$$\frac{ax+b}{cx+d}$$
   where $ad - bc \neq 0$.

3. Let G be as in the last question with $E = \mathbb{Q}$, and let H be the subgroup generated by $x \mapsto x + 1$ and $x \mapsto -1/x$. Find $|H|$ and find f such that $Fix(H) = \mathbb{Q}(f)$

4. Compute $\Gamma(\mathbb{R}/\mathbb{Q})$.

5. (Optional) Repeat the work of problem 1 for the polynomial $x^5 - 2$.