

1. Let $\{a_1, a_2, a_3, \dots, a_n\}$ be a set of positive real numbers. Set

$$S = \sum_{i,j=1}^n \frac{a_i}{a_j}.$$

Find the best upper and lower bounds for this sum that you can. You may want to express your bounds in terms of

$$\begin{aligned} A &= \sum_{i=1}^n a_i \\ \alpha &= \prod_{i=1}^n a_i \\ n & \end{aligned}$$

For example $S \geq n$. Expressions in terms of other useful quantities are also accepted.

2. A deck of 52 playing cards contains 13 trump cards. Four hands of 13 cards are dealt, one of which is yours. Your hand contains k trump cards. What is the expected number of trump cards in each of the other three hands? In the other three hands, what is the expected number of trumps in the hand with the *largest* number of trumps?

Repeat the problem for a deck of 78 cards with 21 trump cards, where three, four or five hands are dealt. Analyze the situation when the deck has n cards, p of which are trumps, and m hands are dealt.