Lecture 6

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1.

$$\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}dx = \lim_{t\to\infty}\int_0^t \frac{x^2}{\sqrt{1+x^3}}dx = \lim_{t\to\infty}\frac{1}{3}\int_1^{1+t^2}\frac{1}{\sqrt{u}}du = \lim_{t\to\infty}\left[\frac{2}{3}\sqrt{u}\right]_1^{1+t^3} = \lim_{t\to\infty}\frac{2}{3}\sqrt{1+t^3}-\frac{2}{3} = \infty$$

using $u = 1+x^3$, $du = 3x^2dx$.

2.

$$\int_1^\infty \frac{1}{x^2+x}dx = \int_1^\infty \frac{1}{x(x+1)}dx = \int_1^\infty \frac{1}{x}+\frac{-1}{x+1}dx = \lim_{t\to\infty}[\ln|x|-\ln|x+1|]_1^t$$
$$= \lim_{t\to\infty}[\ln|t|-\ln|t+1|+\ln 2] = \lim_{t\to\infty}\ln\left|\frac{t}{t+1}\right|+\ln 2 = \ln 2$$

3.

$$\int_{-\infty}^\infty x^3e^{-x^4}dx = \int_0^\infty x^3e^{-x^4}dx + \int_{-\infty}^0 x^3e^{-x^4}dx = \lim_{t\to\infty}[-\frac{1}{4}e^{-x^4}]_0^t + \lim_{s\to-\infty}[-\frac{1}{4}e^{-x^4}]_s^0$$
$$= \lim_{t\to\infty}[-\frac{1}{4}e^{-t^4}+\frac{1}{4}] + \lim_{s\to-\infty}[-\frac{1}{4}+\frac{1}{4}e^{-s^4}] = \frac{1}{4}-\frac{1}{4} = 0$$

4.

$$\int_{-2}^3 \frac{1}{x^4}dx = \int_{-2}^0 \frac{1}{x^4}dx + \int_0^3 \frac{1}{x^4}dx = \lim_{t\to 0^-}\int_{-2}^t \frac{1}{x^4}dx + \lim_{s\to 0^+}\int_s^3 \frac{1}{x^4}$$
$$= \lim_{t\to 0^-}[-\frac{1}{3}x^{-3}]_{-2}^t + \lim_{s\to 0^+}[-\frac{1}{3}x^{-3}]_s^3$$
$$= \lim_{t\to 0^-}[-\frac{1}{3}t^{-3}+\frac{1}{3}(-2)^{-3}] + \lim_{s\to 0^+}[-\frac{1}{3}3^{-3}+\frac{1}{3}s^{-3}]$$
$$= \infty$$

notice the function we are integrating is not defined at $x = 0$.

5.

$$\int_0^3 \frac{dx}{x^2-6x+5} = \int_0^3 \frac{dx}{(x-5)(x-1)} = \int_0^1 \frac{dx}{(x-5)(x-1)} + \int_1^3 \frac{dx}{(x-5)(x-1)}$$
$$= \lim_{t\to 1^-}\int_0^t \frac{dx}{(x-5)(x-1)} + \lim_{s\to 1^+}\int_s^3 \frac{dx}{(x-5)(x-1)}$$
$$= \lim_{t\to 1^-}\int_0^t \frac{1/4}{x-5}+\frac{-1/4}{x-1}dx + \lim_{s\to 1^+}\int_s^3 \frac{1/4}{x-5}+\frac{-1/4}{x-1}dx$$
$$= \lim_{t\to 1^-}\left[\frac{1}{4}\ln|x-5|-\frac{1}{4}\ln|x-1|\right]_0^t + \lim_{s\to 1^+}\left[\frac{1}{4}\ln|x-5|-\frac{1}{4}\ln|x-1|\right]_s^0$$
$$= \frac{1}{4}(\ln 4 - \lim_{t\to 1^-}\ln|x-t| - \ln 5 + \ln 1 + \ln 5 - \ln 1 - \ln 4 + \lim_{s\to 1^+}\ln|s-1|)$$
$$= \text{divergent}$$