Lecture 2

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1.

$$\int \cos^2(x)\tan^3(x)\ dx = \int \cos^2(x)(\sec^2(x)-1)\tan(x)\ dx = \int \tan(x) - \cos(x)\sin(x)dx$$
$$= \int \tan(x) - \frac{1}{2}\sin(2x)dx = -\ln|\cos(x)| + \frac{1}{4}\cos(2x) + C$$

2. Consider $u = \sec(x)$ then $du = \sec(x)\tan(x)dx$, so

$$\int \tan(x)\sec^3(x)dx = \int u^2 du = \frac{u^3}{3} + C = \frac{\sec^3(x)}{3} + C$$

3.

$$\int \tan^3(x)\sec(x)dx = \int \tan(x)(\sec^2(x)-1)\sec(x)dx$$
$$= \int \tan(x)\sec^3(x) - \sec(x)\tan(x)\ dx$$
$$= \frac{1}{3}\sec^3(x) - \sec(x) + C$$

4. Note that $|x| \leq 1$ for the function to be real valued. Thus let $x = \sin\theta$, then $dx = \cos\theta d\theta$ so

$$\int x^3\sqrt{1-x^2}dx = \int \sin^3\theta\cos\theta\cos\theta d\theta$$
$$= \int \sin\theta(1-\cos^2\theta)\cos^2\theta d\theta$$
$$= \int \sin\theta\cos^2\theta - \sin\theta\cos^4\theta\ d\theta$$
$$= -\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5} + C$$
$$= -\frac{(1-x^2)^{3/2}}{3} + \frac{(1-x^2)^{5/2}}{5} + C$$

5. Note that $|x| \leq \frac{1}{2}$ so let $x = \frac{1}{2}\sin\theta$ so $dx = \frac{1}{2}\cos\theta d\theta$, so

$$\int \sqrt{1-4x^2}dx = \int \frac{1}{2}\sqrt{1-\sin^2\theta}\cos\theta d\theta$$
$$= \frac{1}{2}\int \cos^2\theta d\theta$$
$$= \frac{1}{4}\int (\cos(2\theta)+1)d\theta$$
$$= \frac{1}{8}\sin(2\theta) + \frac{\theta}{4} + C$$
$$= \frac{2}{8}\sin\theta\cos\theta + \frac{\theta}{4} + C$$
$$= x\sqrt{\frac{1}{4} - x^2} + \frac{1}{4}\sin^{-1}(2x) + C$$

6. Let $x = 3\sec\theta$, then $dx = 3\sec\theta\tan\theta d\theta$. Note $\theta = \cos^{-1}(\frac{3}{x})$

$$\begin{aligned}
\int \frac{\sqrt{x^2-9}}{x^3}dx &= \int \frac{3\tan\theta}{27\sec^3\theta} 3\sec\theta\tan\theta d\theta \\
&= \frac{9}{27}\int \frac{\tan^2\theta}{\sec^2\theta} d\theta \\
&= \frac{1}{3}\int \sin^2\theta d\theta \\
&= \frac{1}{3}(\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)) + C \\
&= \frac{1}{3}(\frac{1}{2}\theta - \frac{1}{2}\sin(\theta)\cos(\theta)) + C \\
&= \frac{1}{6}(\cos^{-1}(\frac{3}{x}) - \sin(\cos^{-1}(\frac{3}{x}))\cos(\cos^{-1}(\frac{3}{x}))) + C \\
&= \frac{1}{6}(\cos^{-1}(\frac{3}{x}) - \sqrt{1 - \frac{9}{x^2}}\frac{3}{x}) + C \\
&= \frac{1}{6}(\cos^{-1}(\frac{3}{x}) - \frac{3}{x}\sqrt{1 - \frac{9}{x^2}}) + C
\end{aligned}$$

7. Let $x = \frac{3}{5}\sin\theta$, $dx = \frac{3}{5}\cos\theta d\theta$. Note $0.6 = \frac{3}{5} = \frac{3}{5}\sin(\pi/2)$ and $0 = \frac{3}{5}\sin(0)$. Thus we are in the first quadrant, so $\cos\theta = \sqrt{1-\sin\theta}$

$$\begin{aligned}
\int_0^{0.6} \frac{x^2}{\sqrt{9-25x^2}}dx &= \int_0^{0.6} \frac{x^2}{5\sqrt{\frac{9}{25} - x^2}}dx \\
&= \frac{1}{5}\int_0^{\pi/2} \frac{(\frac{3}{5}\sin\theta)^2}{\frac{3}{5}\cos\theta}\frac{3}{5}\cos\theta d\theta \\
&= \frac{9}{125}\int_0^{\pi/2} \sin^2\theta d\theta \\
&= \frac{9}{125}\left[\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\right]_0^{\pi/2} \\
&= \frac{9\pi}{500}
\end{aligned}$$