

21-111 Calculus I - Fall 2004

Review 1

September 20, 2004

1. Simplify the following leaving no complex fractions and negative exponents. Also simplify the radicals as far as possible.

(a) $\frac{x^{\frac{4}{3}}y^{\frac{2}{3}}}{(xy)^{\frac{1}{3}}}$ **Solution:** $xy^{\frac{1}{3}}$

(b) $\frac{\frac{y-x}{xy} - \frac{x-y}{xy}}{\frac{x+y}{x+y}}$ **Solution:** $-(x^2 - y^2) \left(\frac{x+y+xy}{(xy)^2} \right)$

(c) $\frac{\frac{x+y}{xy-x} + \frac{y}{x}}{\frac{x+y}{xy-x} + \frac{x}{y}}$ **Solution:** $\frac{x^2+y}{xy^2-xy+x^2}$

(d) $\left(\frac{x^{-2}y}{16x^2y^2z^{-2}} \right)^{-\frac{3}{2}}$ **Solution:** $\frac{64x^6y^{\frac{3}{2}}}{z^3}$

2. Solve the following equations for x

(a) $\sqrt{x} - 9 = 0$ **Solution:** $x = 81$

(b) $x - 3\sqrt{x} + 2 = 0$ **Solution:** $x = 1$ and $x = 4$

(c) Changed slightly: $4x^2 + 16x + 3 = 0$ **Solution:** $x = -2 \pm \frac{1}{2}\sqrt{13}$

(d) $xyz - 2 = z^2 + a$ **Solution:** $\frac{z^2+a-2}{yz}$

(e) Slightly changed: $3x^3 - x^2 - 6x + 2 = 0$ **Solution:** $x = \pm\sqrt{2}$ and $x = \frac{1}{3}$

(f) $\frac{1}{x-1} + \frac{1}{x+1} = \frac{3}{x^2-1}$ **Solution:** $x = \frac{3}{2}$

(g) $2x^3 + 3x^2 - 3x - 2 = 0$ **Solution:** $x = 1$, $x = -2$ or $x = -\frac{1}{2}$

3. Let $f(x) = \frac{1}{1+x}$ for $x \neq -1$ and let $g(x) = \frac{3x}{1+x^2}$ for $x > 1$. Find the functions $h(x)$ and their domains:
- (a) $h(x) = f(x) + g(x)$ **Solution:** $h(x) = \frac{4x^2+3x+1}{(1+x)(1+x^2)}$ for $x > 1$
 - (b) $h(x) = \frac{f(x)}{g(x)}$ **Solution:** $h(x) = \frac{1+x^2}{(3x+3x^2)}$ for $x > 1$
 - (c) $f \circ g$ **Solution:** $h(x) = \frac{1+x^2}{1+x^2+3x}$ for $x > 1$ and $x \neq \frac{3+\sqrt{5}}{2}$
 - (d) Slightly changed: $g(f(x))$ **Solution:** $h(x) = \frac{3+3x}{x^2+2x+2}$, but as we need $f(x) > 1$ (as domain of g is $x > 1$) we get $-1 < x < 0$ as the domain (Note that for all other values for x , $f(x) < 1$)
4. It costs a company \$ 70 to produce a product. The fixed costs of production are \$ 1000. The revenue of the company is $R(x) = x^2 - 750x + 1000$ for x items sold.
- (a) Find the cost to produce x items. Find out how many items the company has to produce and sell to come out even.
Solution: $C(x) = 70x + 1000$, $Profit(x) = x^2 - 820x = 0$ for $x = 0$ and $x = 820$
 - (b) The company produces $P(t) = 4t$ products per hour. Find the function representing the profit made by the company after producing for t hours. What is the initial cost and what time is needed to have no loss.
Solution: $Profit(P(t)) = 16t^2 - 3280t$ so there is no initial cost and the company needs to produce and sell for 205 hours.
5. Find the zeros of $f(x) = x^3 - x^2 - 3x + 3$ and $g(x) = \frac{|x^2-3|}{2x}$.
- Solution:** The zeros of $f(x)$ are $x = \pm\sqrt{3}$ and $x = 1$ and the zeros of $g(x)$ are $x = \pm\sqrt{3}$.
6. A family starts a college fund for their kids by putting \$ 10,000 into an account for three years running. The account has 4% interest per year compounded quarterly. How much does the family have after 18 years. [Hint: $(1.01)^{72} \approx 2.0471$, $(1.01)^{68} \approx 1.9672$, and $(1.01)^{64} \approx 1.8905$]
- Solution:** \$ 59,048