# ERRATUM FOR "COHOMOLOGIES OF TAUTOLOGICAL BUNDLES OF MATROIDS"

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All theorem/proposition numbers refer to [Eur24] unless otherwise indicated. The proof of Theorems 1.5 and 1.6, which were used to deduce Theorems 1.2 and 1.3, contained a gap, pointed out to us by Matt Larson and Ruizhen Liu. In Section 1, we explain the gap. In Section 2, we briefly describe a workaround given by [Liu] that salvages Theorems 1.2 and 1.3 and establishes a generalization of the theorem with new consequences. In Section 3, we explain how to salvage Theorems 1.5 and 1.6 (which [Liu] does not salvage). All statements in [Eur24] are hence salvaged.

We follow the notations and conventions as laid out in [Eur24]. Throughout, we assume that $n$ is neither a loop nor a coloop in the matroid $\mathrm{M}$ of $L \subseteq \Bbbk^E$, since that is the case in which the gap is nontrivial.

## 1. The gap

The gap arises from the ambiguity of the phrase "natural isomorphism" in Propositions 3.4 and 3.5. Let us explain the intended meaning, and why the propositions fail to establish them.

From the short exact sequence $0 \to \mathcal{S}_L \to \mathcal{O}_{X_E}^{\oplus E} \to \mathcal{Q}_L \to 0$, we have the maps $\bigwedge^p \mathcal{S}_L \hookrightarrow \bigwedge^p \mathcal{O}_{X_E}^{\oplus E}$ and $\bigwedge^p \mathcal{O}_{X_E}^{\oplus E} \twoheadrightarrow \bigwedge^p \mathcal{Q}_L$. The argument given in [Eur24] for the vanishing of the higher direct images of $\bigwedge^p \mathcal{S}_L$ and $\bigwedge^p \mathcal{Q}_L$ under the map $f : X_E \to X_{E\backslash n}$ remains valid. Moreover, because the kernel of $\bigwedge^p \mathcal{O}_{X_E}^{\oplus E} \twoheadrightarrow \bigwedge^p \mathcal{Q}_L$ has a filtration whose successive quotients are $\bigwedge^i \mathcal{S}_L \otimes \bigwedge^j \mathcal{Q}_L$, a modification of the argument (see, for example, [Liu, Lemma 3.7(i)]) yields that the higher direct images of the kernel also vanish. As $f$ is flat and satisfies $Rf_*\mathcal{O}_{X_E} = \mathcal{O}_{X_{E\backslash n}}$, the vanishing of the higher direct images along with Grauert's theorem implies that we have an injection (resp. surjection) of vector bundles $f_*\bigwedge^p \mathcal{S}_L \hookrightarrow \bigwedge^p \mathcal{O}_{X_{E\backslash n}}^{\oplus E}$ (resp. $\bigwedge^p \mathcal{O}_{X_{E\backslash n}}^{\oplus E} \twoheadrightarrow f_*\bigwedge^p \mathcal{Q}_L$). By $(\Bbbk^*)^{E\backslash n}$-equivariance, one can then conclude Theorem 1.5 if the following holds:

> Under the identification $\bigwedge^p \mathcal{O}_{X_{E\backslash n}}^{\oplus E}|_{\mathbf{1}} = \bigwedge^p \Bbbk^E$ of the fiber of $\bigwedge^p \mathcal{O}_{X_{E\backslash n}}^{\oplus E}$ over the identity $\mathbf{1}$ in the open dense torus $(\Bbbk^*)^{E\backslash n}/\Bbbk^* \subset X_{E\backslash n}$, the fiber $f_*\bigwedge^p \mathcal{S}_L|_{\mathbf{1}}$ (resp. $f_*\bigwedge^p \mathcal{Q}_L|_{\mathbf{1}}$) coincides with that of the subbundle (resp. quotient bundle) $\bigwedge^p \mathcal{S}_{L/n}$ (resp. $\bigwedge^p \mathcal{Q}_{L/n} \oplus \bigwedge^{p-1} \mathcal{Q}_{L\backslash n}$) of $\bigwedge^p \mathcal{O}_{X_{E\backslash n}}^{\oplus E}$.

By cohomology and base change, the analysis of the fiber reduces to analyzing $H^0$ of the bundle denoted $\bigwedge^p \mathcal{S}'_L$ (resp. $\bigwedge^p \mathcal{Q}'_L$). By "natural isomorphism" in Proposition 3.4, the intended implication was that the computation given for $H^0(\bigwedge^p \mathcal{S}'_L)$ (resp. $H^0(\bigwedge^p \mathcal{Q}'_L)$) as a subspace (resp. a quotient space) of $\bigwedge^p \Bbbk^E$ is exactly the result of applying $H^0$ to the injection $\bigwedge^p \mathcal{S}'_L \hookrightarrow \bigwedge^p \mathcal{O}_{\mathbb{P}^1}^{\oplus E}$ (resp. surjection $\bigwedge^p \mathcal{O}_{\mathbb{P}^1}^{\oplus E} \twoheadrightarrow \bigwedge^p \mathcal{Q}'_L$). The proof of Proposition 3.4 however does not do this. Using

the diagrams of short exact sequences in the proof of Proposition 3.2, one can deduce straightforwardly the intended implication for $\bigwedge^p \mathcal{S}_L$ and $\mathcal{Q}_L$. However, the case of $\bigwedge^p \mathcal{Q}_L$ requires more work when $p > 1$, as described in Section 3 below. The argument for Theorem 1.6 suffers a related but different gap. First, whether the map $\operatorname{Sym}^p \mathcal{O}_{X_{E\setminus n}} \to f_* \operatorname{Sym}^p \mathcal{Q}_L$ is surjective is unclear, since the successive quotients of the filtration for $\operatorname{Sym}^p \mathcal{O}_{X_E} \to \operatorname{Sym}^p \mathcal{Q}_L$ are $\operatorname{Sym}^i \mathcal{S}_L \otimes \operatorname{Sym}^{p-i} \mathcal{Q}_L$, which can have nonvanishing $R^1 f_*$. Second, Proposition 3.5 only computes the fibers of the bundle $f_* \operatorname{Sym}^p \mathcal{Q}_L$ with no indication that the fiberwise computations glue to give the desired isomorphism $f_* \operatorname{Sym}^p \mathcal{Q}_L \simeq \operatorname{Sym}^p(\mathcal{Q}_{L/n} \oplus \mathcal{O}_{E\setminus n})$. Therefore, the proof of the formulas for $f_*(\bigwedge^p \mathcal{Q}_L)$ and $f_*(\operatorname{Sym}^p \mathcal{Q}_L)$ in Theorems 1.5 and 1.6 when $p > 1$ must be considered incomplete.

## 2. A REMEDY

Theorems 1.2 and 1.3 are salvaged exceedingly well in [Liu], in the sense that the author shows more generally the higher cohomology vanishing for $\bigwedge^p(\mathcal{S}_L \oplus \mathcal{Q}_L)$ [Liu, Theorem B] and for the Schur powers $\mathbb{S}^\lambda \mathcal{Q}_L$ for any hook-shape partition $\lambda$ [Liu, Proposition 3.17]. The author achieves [Liu, Theorem B] via a clever induction on $n$ which only requires the computation of $f_* \bigwedge^p \mathcal{S}_L$ and $f_* \bigwedge^p \mathcal{Q}_L^\vee$, and which avoids the need for the formula for $f_* \bigwedge^p \mathcal{Q}_L$ claimed in Theorem 1.5.

A new observation in [Liu] that streamlines the computation of $f_* \bigwedge^p \mathcal{S}_L$ and $f_* \bigwedge^p \mathcal{Q}_L^\vee$, as well as facilitates the induction, is the following. An injection $K \hookrightarrow L$ of subspaces of $\Bbbk^E$ gives rise to the short exact sequence $0 \to \mathcal{S}_K \to \mathcal{S}_L \to \mathcal{S}_L/\mathcal{S}_K \to 0$ of vector bundles (and similarly for $\mathcal{Q}$). When the injection is $L/n \oplus 0 \hookrightarrow L$, the resulting sequence specializes to $0 \to f^* \mathcal{S}_{L/n} \to \mathcal{S}_L \to \mathcal{L}_1 \to 0$, with the property that over fibers of $f$, it restricts to become the sequence $0 \to \mathcal{S}'_{L/n\oplus 0} \to \mathcal{S}'_L \to \mathcal{L}_\mathcal{S} \to 0$ appearing in Lemma 3.2. When the injection is $L/n \hookrightarrow L \setminus n$, the resulting sequence can be used to facilitate the induction.

## 3. THE FORMULAS FOR $f_* \bigwedge^p \mathcal{Q}_L$ AND $f_* \operatorname{Sym}^p \mathcal{Q}_L$

We describe how to salvage the formulas $f_* \bigwedge^p \mathcal{Q}_L = \bigwedge^p \mathcal{Q}_{L/n} \oplus \bigwedge^{p-1} \mathcal{Q}_{L\setminus n}$ and $f_* \operatorname{Sym}^p \mathcal{Q}_L \simeq \operatorname{Sym}^p(\mathcal{Q}_{L/n} \oplus \mathcal{O}_{X_{E\setminus n}})$, claimed in Theorems 1.5 and 1.6 (respectively). These therefore salvage Theorems 1.2 and 1.3 without appealing to [Liu, Theorem B and Proposition 3.7].

3.1. **Exterior power case.** Per Section 1, we need to verify that, under the canonical identifications

$$H^0(\textstyle\bigwedge^p \mathcal{O}_{\mathbb{P}^1}^{\oplus E}) \simeq \bigwedge^p \Bbbk^E \simeq \bigwedge^p \Bbbk^{E\setminus n} \oplus \bigwedge^{p-1} \Bbbk^{E\setminus n} \otimes \Bbbk^{\{n\}},$$

the map $\varphi : H^0(\bigwedge^p \mathcal{O}_{\mathbb{P}^1}^{\oplus E}) \to H^0(\bigwedge^p \mathcal{Q}'_L)$ is surjective with the kernel equal to the kernel of the map

$$\psi : \textstyle\bigwedge^p \Bbbk^{E\setminus n} \oplus \bigwedge^{p-1} \Bbbk^{E\setminus n} \otimes \Bbbk^{\{n\}} \to \bigwedge^p \big(\Bbbk^{E\setminus n}/(L/n)\big) \oplus \bigwedge^{p-1} \big(\Bbbk^{E\setminus n}/(L\setminus n)\big) \otimes \Bbbk^{\{n\}}.$$

The map $\varphi$ is surjective because the kernel of $\bigwedge^p \mathcal{O}_{\mathbb{P}^1}^{\oplus E} \twoheadrightarrow \bigwedge^p \mathcal{Q}'_L$ has a filtration whose successive quotients are $\bigwedge^i \mathcal{S}'_L \otimes \bigwedge^{p-i} \mathcal{Q}'_L$, each of which has vanishing $H^1$ [Liu, Lemma 3.4(i)]. Moreover, the proof of Proposition 3.4 shows that the dimensions of the two spaces $H^0(\bigwedge^p \mathcal{Q}'_L)$ and $\bigwedge^p \big(\Bbbk^{E\setminus n}/(L/n)\big) \oplus \bigwedge^{p-1} \big(\Bbbk^{E\setminus n}/(L\setminus n)\big) \otimes \Bbbk^{\{n\}}$ are equal (although it fails to show that they are "naturally isomorphic" in the desired sense). It therefore suffices now to show that $\ker \varphi \subseteq \ker \psi$.

Suppose $v \in \bigwedge^p \Bbbk^E$, considered as a constant global section of $\bigwedge^p \mathcal{O}_{\mathbb{P}^1}^{\oplus E}$, satisfies $\varphi(v) = 0$. By the definition of $\mathcal{Q}'_L$, the fiber over $t \in \Bbbk^* \subset \mathbb{P}^1$ is $\bigwedge^p\big(\Bbbk^E/(t \cdot L)\big)$, where we recall that $\Bbbk^*$ acts on $\Bbbk^E$ by $t \cdot (x_1, \ldots, x_n) = (x_1, \ldots, x_{n_1}, t^{-1}x_n)$. Then, letting $\widetilde{\varphi}$ be the map $\bigwedge^p \Bbbk^E \twoheadrightarrow \bigwedge^p(\Bbbk^E/L)$, we find that the condition $\varphi(v) = 0$ implies $\widetilde{\varphi}(t^{-1} \cdot v) = 0$ for all $t \in \Bbbk^*$. Let $\pi_1$ and $\pi_2$ be the projection to the two summands of $\bigwedge^p \Bbbk^{E\setminus n} \oplus \bigwedge^{p-1} \Bbbk^{E\setminus n} \otimes \Bbbk^{\{n\}}$ (respectively). Then, using the standard basis for $\bigwedge^p \Bbbk^E$, one verifies straightforwardly that $\lim_{t\to 0}(t^{-1} \cdot v) = \pi_1(v)$ and $\lim_{t\to 0} \frac{1}{t}(t^{-1} \cdot' v) = \pi_2(v)$. Thus, we have that both $\pi_1(v)$ and $\pi_2(v)$ belong to $\ker \widetilde{\varphi}$. From this, we show that both $\pi_1(v)$ and $\pi_2(v)$ belong to $\ker \psi$, and thereby $v \in \ker \psi$, as follows.

The injection of short exact sequences

$$\begin{array}{ccccccccc}
0 & \longrightarrow & L/n & \longrightarrow & \Bbbk^{E\setminus n} & \longrightarrow & \Bbbk^{E\setminus n}/(L/n) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & L & \longrightarrow & \Bbbk^E & \longrightarrow & \Bbbk^E/L & \longrightarrow & 0
\end{array}$$

induces a commuting diagram

$$\begin{array}{ccc}
\bigwedge^p \Bbbk^{E\setminus n} & \twoheadrightarrow & \bigwedge^p\big(\Bbbk^{E\setminus n}/(L/n)\big) \\
\downarrow & & \downarrow \\
\bigwedge^p \Bbbk^E & \xrightarrow{\widetilde{\varphi}} \twoheadrightarrow & \bigwedge^p(\Bbbk^E/L),
\end{array}$$

from which we conclude that $\pi_1(v) \in \ker \widetilde{\varphi}$ implies $\pi_1(v) \in \ker \psi$. Similarly, the bottom two rows of the commuting diagram of short exact sequences (where $L \simeq L \setminus n$ because $n$ is not a coloop)

$$\begin{array}{ccccccccc}
 & & & & 0 & & 0 & & \\
 & & & & \downarrow & & \downarrow & & \\
 & & 0 & \longrightarrow & L & \xrightarrow{\simeq} & L \setminus n & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Bbbk^{\{n\}} & \longrightarrow & \Bbbk^E & \longrightarrow & \Bbbk^{E\setminus n} & \longrightarrow & 0 \\
 & & \| & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Bbbk^{\{n\}} & \longrightarrow & \Bbbk^E/L & \longrightarrow & \Bbbk^{E\setminus n}/(L \setminus n) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

induces a commuting diagram of short exact sequences

$$\begin{array}{ccccccccc}
0 & \longrightarrow & \bigwedge^{p-1} \Bbbk^{E\setminus n} \otimes \Bbbk^{\{n\}} & \longrightarrow & \bigwedge^p \Bbbk^E & \longrightarrow & \bigwedge^p \Bbbk^{E\setminus n} & \longrightarrow & 0 \\
 & & \downarrow & & \widetilde{\varphi}\downarrow & & \downarrow & & \\
0 & \longrightarrow & \bigwedge^{p-1}\big(\Bbbk^{E\setminus n}/(L \setminus n)\big) \otimes \Bbbk^{\{n\}} & \longrightarrow & \bigwedge^p(\Bbbk^E/L) & \longrightarrow & \bigwedge^p\big(\Bbbk^{E\setminus n}/(L \setminus n)\big) & \longrightarrow & 0
\end{array}$$

from which we conclude that $\pi_2(v) \in \ker \widetilde{\varphi}$ implies $\pi_2(v) \in \ker \psi$.

3.2. **Symmetric power case.** As discussed in Section 1, the map $\mathrm{Sym}^p \mathcal{O}_{X_{E\setminus n}} \to f_* \mathrm{Sym}^p \mathcal{Q}_L$ may not be surjective, and hence the symmetric power case requires a proof different from the one we have given above for the exterior power case. We provide a proof utilizing notions introduced in [Liu]. Let us adopt the notations $\mathcal{L}_1$, $\mathcal{N}_1$, and $\mathcal{L}_2$ as described in [Liu, (3.2.1), (3.2.3), (3.3.1)]. From the chain of injections $L/n \oplus 0 \hookrightarrow L \hookrightarrow L \setminus n \oplus L|n$, one obtains the following commuting diagram of short exact sequences, when one notes that $\mathcal{Q}_{L/n\oplus 0} = f^*\mathcal{Q}_{L/n} \oplus (\mathcal{O}_{X_E} \otimes L|n)$ and $\mathcal{Q}_{L\setminus n\oplus L|n} = f^*\mathcal{Q}_{L\setminus n}$ since $L|n = \Bbbk^{\{n\}}$ (as $n$ is not a loop by assumption):

$$
(\dagger) \qquad
\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \mathcal{L}_1 & \longrightarrow & f^*\mathcal{L}_2 \oplus (\mathcal{O}_{X_E} \otimes L|n) & \longrightarrow & \mathcal{N}_1 & \longrightarrow & 0 \\
 & & \| & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \mathcal{L}_1 & \longrightarrow & f^*\mathcal{Q}_{L/n} \oplus (\mathcal{O}_{X_E} \otimes L|n) & \longrightarrow & \mathcal{Q}_L & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & \longrightarrow & f^*\mathcal{Q}_{L\setminus n} & = & f^*\mathcal{Q}_{L\setminus n} & \longrightarrow & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array}
$$

Taking the natural filtrations for $\mathrm{Sym}^p$ of $f^*\mathcal{Q}_{L/n} \oplus (\mathcal{O}_{X_E} \otimes L|n)$ and $\mathcal{Q}_L$ given by the right two vertical short exact sequences, and then applying $f_*$ along with the projection formula, we obtain a map of filtrations

$$
\begin{array}{ccccccc}
\mathcal{E}_p & \hookrightarrow & \mathcal{E}_{p-1} & \hookrightarrow & \cdots & \hookrightarrow & \mathcal{E}_0 = \mathrm{Sym}^p\left(\mathcal{Q}_{L/n} \oplus (\mathcal{O}_{X_{E\setminus n}} \otimes L|n)\right) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
f_*\mathcal{F}_p & \hookrightarrow & f_*\mathcal{F}_{p-1} & \hookrightarrow & \cdots & \hookrightarrow & f_*\mathcal{F}_0 = f_* \mathrm{Sym}^p \mathcal{Q}_L
\end{array}
$$

where we have $\mathcal{F}_i/\mathcal{F}_{i+1} \simeq \mathcal{N}_1^{\otimes i} \otimes \mathrm{Sym}^{p-i} f^*\mathcal{Q}_{L\setminus n}$ (with $\mathcal{F}_{p+1} = 0$) and similarly for $\mathcal{E}_i/\mathcal{E}_{i+1}$. By an argument similar to the proof of [Liu, Lemma 3.7], which uses that the restriction of $\mathcal{N}_1$ to any component of a fiber of $f$ (which is a chain of $\mathbb{P}^1$'s) is isomorphic to $\mathcal{O}_{\mathbb{P}^1}$ or $\mathcal{O}_{\mathbb{P}^1}(1)$, we have $R^j f_*(\mathcal{N}_1^{\otimes i}) = 0$ for all $i \geq 0$ and $j > 0$. Hence, we have $f_*\mathcal{F}_i/f_*\mathcal{F}_{i+1} \simeq f_*(\mathcal{F}_i/\mathcal{F}_{i+1})$ for all $i$, and by cohomology and base change, for any $x \in X_{E\setminus n}$ the natural map $(f_*\mathcal{F}_i/f_*\mathcal{F}_{i+1})|_x \to H^0(f^{-1}(x), (\mathcal{F}_i/\mathcal{F}_{i+1})|_{f^{-1}(x)})$ is an isomorphism.

We show that the last vertical map is an isomorphism by showing that every map between the successive quotients is an isomorphism. We achieve this by showing that over every point $x \in X_{E\setminus n}$, the map of the fibers $(\mathcal{E}_i/\mathcal{E}_{i+1})|_x \to (f_*\mathcal{F}_i/f_*\mathcal{F}_{i+1})|_x$ is an isomorphism. By $(\Bbbk^*)^{E\setminus n}/\Bbbk^*$-equivariance, or by running the same argument with $t \cdot L_{\mathscr{F}(k)}$ in place of $L$ for ordered partitions $\mathscr{F}$ (as is done in the proof of Theorems 1.5 and 1.6 in [Eur24] or the proof of [Liu, Lemma 3.7]), it suffices to consider only the fiber over the identity point $\mathbf{1} \in (\Bbbk^*)^{E\setminus n}/\Bbbk^* \subset X_{E\setminus n}$.

Restricting the diagram (†) to the fiber $f^{-1}(\mathbf{1}) \simeq \mathbb{P}^1$ yields the diagram

$$
(\ddagger)\qquad
\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \mathcal{L}_{\mathcal{S}} & \longrightarrow & \mathcal{O}_{\mathbb{P}^1} \otimes \big((L\backslash n)/(L/n) \oplus L|n\big) & \longrightarrow & \mathcal{L}_{\mathcal{Q}} & \longrightarrow & 0 \\
 & & \| & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \mathcal{L}_{\mathcal{S}} & \longrightarrow & \mathcal{O}_{\mathbb{P}^1} \otimes \big(\Bbbk^{E\backslash n}/(L/n) \oplus \Bbbk^{\{n\}}\big) & \longrightarrow & \mathcal{Q}'_L & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & \longrightarrow & \mathcal{O}_{\mathbb{P}^1} \otimes \big(\Bbbk^{E\backslash n}/(L\backslash n) \oplus \Bbbk^{\{n\}}/(L|n)\big) & = & \mathcal{Q}'_{L\backslash n \oplus L|n} & \longrightarrow & 0 \\
 & & & & \downarrow & & \downarrow & & \\
 & & & & 0 & & 0 & &
\end{array}
$$

that appears in the proof of Lemma 3.2. Let us denote the vector bundles in the middle column by $\mathcal{U}$, $\mathcal{V}$, and $\mathcal{W}$, from top to bottom. By cohomology and base change, the maps $(\mathcal{E}_i/\mathcal{E}_{i+1})|_{\mathbf{1}} \to (f_*\mathcal{F}_i/\mathcal{F}_{i+1})|_{\mathbf{1}}$ are therefore identified with $H^0$ applied to the maps between the successive quotients of the filtrations of $\operatorname{Sym}^p \mathcal{V}$ and $\operatorname{Sym}^p \mathcal{Q}'_L$, that is,

$$H^0(\operatorname{Sym}^i \mathcal{U} \otimes \operatorname{Sym}^{p-i} \mathcal{W}) \to H^0(\mathcal{L}_{\mathcal{Q}}^{\otimes i} \otimes \operatorname{Sym}^{p-i} \mathcal{W}).$$

This map is an isomorphism since $\mathcal{W}$ is a trivial vector bundle, and since we have $H^0(\operatorname{Sym}^i \mathcal{U}) = \operatorname{Sym}^i \big((L\backslash n)/(L/n) \oplus L|n\big) \simeq H^0(\mathcal{L}_{\mathcal{Q}}^{\otimes i})$ due to the fact that the top row of (‡) is the Euler sequence $0 \to \mathcal{O}_{\mathbb{P}^1}(-1) \to \mathcal{O}_{\mathbb{P}^1}^{\oplus 2} \to \mathcal{O}_{\mathbb{P}^1}(1) \to 0$ by the proof of Lemma 3.2.

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## REFERENCES

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