MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#12

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Problem 1. Consider the linear differential operator $\ell(y) := y'' + 2y' + y$. Find the general solution to $\ell(y) = t\cos t$ as follows:

(a) (2 points) Find $\ker \ell$.

(b) (3 points) Find a particular solution to $\ell(y) = t\cos t$ (Hint: you might end up writing down a 4×4 matrix).

(c) (1 point) Using the previous two parts, state what the general solution to $\ell(y) = t\cos t$ is.

Problem 2. (4 points) Find a particular solution to $y'' - 3y' + 2y = e^{2t}$.

#1. (a) Aux. eq : $r^2+2r+1=0 \Rightarrow (r+1)^2=0$. Thus, $\ker \ell = \operatorname{span}_{\mathbb{R}}(e^{-t}, te^{-t})$

(b) Method of undet. coeff: let $W = \operatorname{span}(\cos t, \sin t, t\cos t, t\sin t)$.

Then $\ell|_W : W \to W$ is represented by:

$$A = \begin{bmatrix} 0 & 2 & 2 & 2 \\ -2 & 0 & -2 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{bmatrix}$$

$(t\cos t)' = \cos t - t\sin t$

$(t\cos t)'' = -2\sin t - t\cos t$

$(t\sin t)' = \sin t + t\cos t$

$(t\sin t)'' = 2\cos t - t\sin t$

Soln to $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \to$ one particular sol: $\frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

$y_p = \frac{1}{2}(\cos t - \sin t + t\sin t)$

(c) Gen soln: $y_p + \ker \ell$, which is $\boxed{\frac{1}{2}(\cos t - \sin t + t\sin t) + C_1 e^{-t} + C_2 te^{-t}}$

#2. Aux. Eq: $r^2 - 3r + 2 = (r-2)(r-1) = 0$. 2 is a root.

Set $y_p = Cte^{2t}$.

$y_p' = Ce^{2t} + 2Cte^{2t}$

$y_p'' = 2Ce^{2t} + 2Ce^{2t} + 4Cte^{2t}$ $\Big] \to \ell(y_p) = Ce^{2t} \Rightarrow C = 1$.

$\therefore \boxed{y_p = te^{2t}}$