MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#8

GSI: CHRISTOPHER EUR, DATE: 4/16/2019

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Problem 1. Solve for all functions $f(x)$ such that

$$f'' - 2f' + 2f = e^x.$$

Problem 2. Let r, s be two distinct real numbers. Show that the two functions $f(t) = e^{rt}, g(t) = e^{st}$ are linearly independent as elements of $C^\infty(\mathbb{R})$.

#1. Guess $f = e^{rx} \rightsquigarrow r^2 - 2r + 2 = 0 \Rightarrow r = 1 \pm \sqrt{-1} = 1 \pm i$

$\Rightarrow$ Soln to homog. eqn $f'' - 2f' + 2f = 0$ is $\text{span}\{e^x \cos x, e^x \sin x\}$.

For the particular soln, guess $f = Ae^x$ (note $(Ae^x)' = Ae^x$)

$\rightsquigarrow Ae^x - \cancel{2Ae^x} + \cancel{2Ae^x} = e^x \Rightarrow A = 1$.

$\therefore \boxed{e^x + c_1 e^x \cos x + c_2 e^x \sin x \quad \forall c_1, c_2 \in \mathbb{R}}$

#2 Suppose lin. dep., then $\exists C \in \mathbb{R}$ st $Ce^{rt} = e^{st}$.

But then $C = e^{(s-r)t}$.

As $s \neq r$, $e^{(s-r)t}$ is not a constant function

$\left(e^{(s-r)0} = 1, \quad e^{(s-r)\frac{1}{s-r}} = e^1 \neq 1 \right)$

Thus we cannot have had $C = e^{(s-r)t}$ $\rightsquigarrow$ contradiction.

$\therefore$ e^{rt}, e^{st} lin. indep.