MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#5

GSI: CHRISTOPHER EUR, DATE: 3/12/2019

STUDENT NAME: Spring Forward

Problem 1. Define $T : \mathbb{P}_3 \to \mathbb{R}^4$ given by $T(p(x)) = \begin{bmatrix} p(-3) \\ p(-1) \\ p(1) \\ p(3) \end{bmatrix}$. Let $B = \{1, x, x^2, x^3\}$ and $E = \{e_1, e_2, e_3, e_4\}$ be bases for $\mathbb{P}_3$ and $\mathbb{R}^4$ (respectively). Find the matrix of transformation ${}_E[T]_B$.

Problem 2. True/false: if true, give a justification; if false, give a counterexample.

(1) If A is diagonalizable $n \times n$ matrix, then A has n distinct eigenvalues.
(2) If A is invertible, then A is diagonalizable.

(1) $T(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $T(x) = \begin{bmatrix} -3 \\ -1 \\ 1 \\ 3 \end{bmatrix}$, $T(x^2) = \begin{bmatrix} 9 \\ 1 \\ 1 \\ 9 \end{bmatrix}$, $T(x^3) = \begin{bmatrix} -27 \\ -1 \\ 1 \\ 27 \end{bmatrix}$

$${}_E[T]_B = \begin{bmatrix} 1 & -3 & 9 & -27 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \end{bmatrix}$$

(2) (a) False: Consider $\begin{bmatrix} 2 & & 0 \\ & \ddots & \\ 0 & & 2 \end{bmatrix}$.

(b) False: Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$. A is invtbl. ($\det A = 4 \neq 0$)
But char pol. A is $(2-\lambda)^2$, so that $\lambda = 2$ is the only eigenval. But $\dim \operatorname{nul}(A - 2I) = 1$, so that there is only one eigenvec. of A (upto scaling).
$\therefore$ A not diagonalizable.