MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#10

GSI: CHRISTOPHER EUR, DATE: 4/30/2019

STUDENT NAME: Dothraki why ?!

For both of the problems, let A be a matrix

$$\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}.$$

Problem 1. Find the fundamental matrix $X(t)$ of $\mathbf{x}'(t) = A\mathbf{x}(t)$. (That is, a matrix $X(t)$ whose columns form a basis for the solution space of $\mathbf{x}'(t) = A\mathbf{x}(t)$). And compute the inverse $X^{-1}(t)$.

Problem 2. Now, compute the general solution to the inhomogeneous equation $\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{f}(t)$, where $\mathbf{f}(t) = \begin{bmatrix} 2e^t \\ 4e^t \end{bmatrix}$.

#1. Char pol A : $\lambda^2 - 1 = 0 \Rightarrow$ eigenval: $1, \quad -1$

eigenvec: $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$$X(t) = \begin{bmatrix} e^t & -e^{-t} \\ -e^t & 3e^{-t} \end{bmatrix}, \qquad X^{-1}(t) = \frac{1}{2}\begin{bmatrix} 3e^{-t} & e^{-t} \\ e^t & e^t \end{bmatrix}$$

#2 particular $\vec{x}$ by

$$\vec{x}(t) = X(t) \int X^{-1}(s)\, f(s)\, ds$$

$$= X(t) \int \begin{bmatrix} 5 \\ 3e^{2s} \end{bmatrix} ds$$

$$= X(t) \begin{bmatrix} 5t \\ \frac{3}{2}e^{2t} \end{bmatrix}$$

$$\therefore \boxed{c_1 \begin{bmatrix} e^t \\ -e^t \end{bmatrix} + c_2 \begin{bmatrix} -e^{-t} \\ 3e^{-t} \end{bmatrix} + \begin{bmatrix} 5te^t - \frac{3}{2}e^t \\ -5te^t + \frac{9}{2}e^t \end{bmatrix}}$$