PRICING, OPTIMALITY, AND EQUILIBRIUM BASED ON COHERENT RISK MEASURES

A.S. Cherny

Abstract. The aim of this paper is to apply the theory of coherent risk measures to the problems of finance.

1. First, we study several problems in the theory of coherent risk measures needed for the applications in finance. In particular,

- we give a simple solution to the problem of the *capital allocation* between several units of a firm;
- this result is applied to introduce the notion of *risk contribution* for coherent risk measures;
- furthermore, this result is applied to the problem of finding the optimal structure of a firm consisting of several units.

2. We consider the pricing technique known as *No Good Deals* and establish the fundamental theorem of asset pricing as well as the form of the fair price intervals. We consider two forms of this technique:

- utility-based pricing (employing the assumption that there is no trade with a negative risk);
- RAROC-based pricing (employing the upper limit on a possible RAROC).

Our general model applies to a wide class of coherent risk measures (satisfying only a compactness condition) and to various financial models, including dynamic ones as well as models with an infinite number of assets (in particular, this allows us to consider models with traded derivatives as basic assets, which makes it possible to narrow considerably fair price intervals). Moreover, the proposed approach takes into account such market imperfections as transaction costs, portfolio constraints, liquidity effects, and ambiguity of a historic probability measure.

3. Next we study the optimization problem based on coherent risk measures. This problem is considered in several setups:

- agent-independent optimization (based on RAROC maximization);
- single-agent global optimization;
- single-agent local optimization.

The results are obtained for a general model and are illustrated by a static model with a finite number of assets, where they admit a simple geometric interpretation.

4. Furthermore, the results described above are applied to the optimality pricing. We present several techniques:

- agent-independent optimality pricing;
- single-agent optimality pricing;
- multi-agent optimality pricing.

The results are obtained for a general model and are illustrated by a static model with a finite number of assets, where they admit a simple geometric interpretation.

5. Finally, we consider the equilibrium problem. We establish the equivalence between different definitions and present a criterion for equilibrium.

Furthermore, the equilibrium technique is applied to pricing. Thus, altogether there are at least eight different pricing techniques based on coherent risk measures that are considered in this paper.