

Interacting Brownian Motions and the Distribution of Equity Capital

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The influence of a stock on the entire equity market is often measured by its market weight, i.e., the proportion of the total market capital that belongs to that stock. Empirical evidence often suggests that the growth and volatility of stock capitals depend less on the name than on the rank of its market weight among the market weights of all stocks. In fact, the distribution curve obtained by plotting ordered log-capitals in American stock exchanges against log of their ranks demonstrates remarkable stability over decades. In order to mathematically model such phenomena, Bob Fernholz, together with his co-authors A. Banner, and I. Karatzas, introduces multivariate interacting geometric Brownian motions whose parameters depend on the random time-varying order in which the coordinates are ranked. Analysis of such rank-based models are difficult, and previous attempts use the equally intractable world of reflecting Brownian motions. We develop novel tools based on polytope geometry to study such processes and their generalizations, and derive long-term properties which are consistent with empirical observations. Our results also settle some of the open questions raised by Banner, Fernholz, and Karatzas. One of the major objectives has been the study of market diversity as implied by the rank-based models. We prove that when the number of stocks in the market is large, most of such models exhibit a phase transition phenomenon which drastically affects the concentration or spread of capital in the equity market.

This is based on a joint work with Sourav Chatterjee of U. C. Berkeley.