

# Exact Solution Approach for Stochastic Portfolio Optimization with Trading Constraints

Miguel Lejeune, Carnegie Mellon University

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In this talk, we first present extensions of the classical mean-variance portfolio optimization model that account for the estimation risk and for trading requirements. We consider that the expected asset returns are stochastic by introducing a probabilistic constraint which imposes that the expected return of the constructed portfolio must exceed a prescribed return threshold with a high confidence level. Depending upon the value of a single parameter, i.e. the quantile of a multivariate probability distribution, we define for which families of probability distributions the deterministic equivalents of the resulting stochastic portfolio optimization models are second-order cone programs, and derive a closed-form approximation of the quantile when its exact value is unknown. We also account for real-world trading constraints, such as the need to diversify the investments in a number of industrial sectors, the non-profitability of holding small positions and the constraint of buying stocks by lots, modeled with integer variables. Further on, we propose an exact solution approach in which the uncertainty in the estimate of the expected returns and the integer trading restrictions are simultaneously considered. The proposed algorithmic approach rests on a non-linear branch-and-bound algorithm which features two new branching rules. The first one is a static rule, called idiosyncratic risk branching, while the second one is dynamic and is called portfolio risk branching. The proposed branching rules are implemented within the open-source solver Bonmin. The comparison of the computational results obtained with standard MINLP solvers and with the proposed approach shows the effectiveness of the latter. Finally, we apply the proposed solution approach to a real-life problem, i.e. the construction of an exchange-traded fund, faced by a major international bank.