The adaptive Poisson disorder problem

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We study the quickest detection problem of a sudden change in the arrival rate of a Poisson process from a known value to an unknown and unobservable value at an unknown and unobservable disorder time. Our objective is to design an alarm time which is adapted to the history of the arrival process and detects the disorder time as soon as possible.

In previous solvable versions of the Poisson disorder problem, the arrival rate after the disorder has been assumed a known constant. In reality, however, we may at most have some prior information on the likely values of the new arrival rate before the disorder actually happens, and insufficient estimates of the new rate after the disorder happens. Consequently, we assume in this paper that the new arrival rate after the disorder is a random variable.

The detection problem is shown to admit a finite-dimensional Markovian sufficient statistic if the new rate has a discrete distribution with finitely-many atoms. Furthermore, the detection problem is cast as a discounted optimal stopping problem with running cost for a finite-dimensional piecewise-deterministic Markov process.

This optimal stopping problem is studied in detail in the special case where the new arrival rate has Bernoulli distribution. This is a non-trivial optimal stopping problem for a two-dimensional piecewise-deterministic Markov process driven by the same point process. Instead of trying to tackle the free boundary problem associated with an integro-differential operator, we solve the problem by constructing an exponentially converging sequence of functions using the iteration of an appropriately defined single-jump operator.