21-112 Calculus II:
Homework #7 Solutions

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1 7.1

Problem 8:
$C(x, y, z) = 3xy + 10yz + 5xz$. The reason for $10yz$ is that there are two yz sides.

Problem 18:
To find out the level curve, one needs to solve first the height of the level curve. Since $(2, \frac{1}{2})$ is a point on the level curve, the height must be 1, $f(2, 1/2) = 2 * 1/2 = 1$. The level curve that we wish to draw is then $1 = xy$, or $y = 1/x$. See Figure 1.

2 7.2

Problem 18:
$f(x, y, z) = ze^{(x+3y)z}$, so:

$$\frac{\partial f}{\partial x} = (ze^{xz+3yz})' = (ze^{3yz}e^{xz})' = z^2e^{3yz}e^{xz} = z^2e^{(x+3y)z}$$
$$\frac{\partial f}{\partial y} = (ze^{xz+3yz})' = (ze^{xz}e^{3yz})' = 3z^2e^{xz}e^{3yz} = 3z^2e^{(x+3y)z}$$
$$\frac{\partial f}{\partial z} = e^{(x+3y)z} + z(x+3y)e^{(x+3y)z} = e^{(x+3y)z}(1 + z(x+3y))$$

Problem 22:

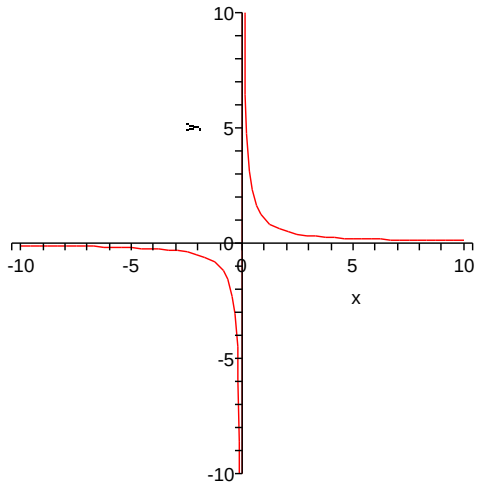


Figure 1: Problem 18 in Section 7.1.

$f(x,y,z) = \frac{x}{y-z}$, so:

$$\frac{\partial f}{\partial y} = (x(y-z)^{-1})' = -x(y-z)^{-2} = -\frac{x}{(y-z)^2}$$

Hence, $\frac{\partial f}{\partial y}(2,-1,3) = -\frac{2}{(-4)^2} = -1/8$.

Problem 24:
$f(x,y,z) = xe^y + x^4y + y^3$, so:

$$\begin{aligned}
\frac{\partial f}{\partial x} &= e^y + 4x^3y \\
\frac{\partial^2 f}{\partial x^2} &= 12x^2y \\
\frac{\partial f}{\partial y} &= xe^y + x^4 + 3y^2 \\
\frac{\partial^2 f}{\partial y^2} &= xe^y + 6y
\end{aligned}$$

$\frac{\partial^2 f}{\partial x \partial y}$ is obtained by differentiating $\frac{\partial f}{\partial y}$ with respect to x, it is $e^y + 4x^3$.
$\frac{\partial^2 f}{\partial y \partial x}$ is obtained by differentiating $\frac{\partial f}{\partial x}$ with respect to y, it is also $e^y + 4x^3$.

Problem 26:

(a) Marginal productivities of labor is computed by differentiating $f(x, y) = 300x^{2/3}y^{1/3}$ with respect to x:

$$\frac{\partial f}{\partial x} = 300 * 2/3 * x^{-1/3} * y^{1/3} = 200x^{-1/3}y^{1/3}$$
$$\frac{\partial f}{\partial x}(125, 64) = 200 * 1/5 * 4 = 160$$

Marginal productivities of capital is computed by differentiating $f(x, y) = 300x^{2/3}y^{1/3}$ with respect to y:

$$\frac{\partial f}{\partial y} = 300 * 1/3 * x^{2/3} * y^{-2/3} = 100x^{2/3}y^{-2/3}$$
$$\frac{\partial f}{\partial y}(125, 64) = 100 * 25/16 = 156.25$$

(b) The problem is asking to compute the difference as the capital is increased from 64 to 66 while labor is fixed at 125. Hence, it is:

$$f(125, 66) - f(125, 64) = f(125, 64 + 2) - f(125, 64) \approx \frac{\partial f}{\partial y}(125, 64) * 2$$
$$\approx 156.25 * 2 = 312.5$$

(c) The problem is asking to compute the difference as the labor is *decreased* from 125 to 124 while capital is fixed at 64. Hence, it is:

$$f(124, 64) - f(125, 64) = f(125 - 1, 64) - f(125, 64) \approx \frac{\partial f}{\partial x}(125, 64) * (-1)$$
$$\approx 160 * (-1) = -160$$

-$156.25 is valid. It indicates that by reducing the labor, the productivity also decreases.