21-112 Calculus II:
Homework #5 Solutions

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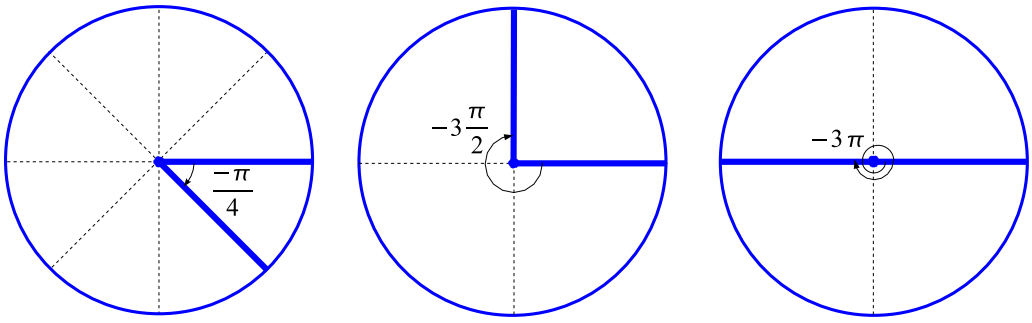
§8.1 #4 Convert the following to radian measure: $990°, -270°, -540°$.

A:

$$990° = 990\left(\frac{\pi}{180}\right) = \frac{11\pi}{2}$$
$$-270° = -270\left(\frac{\pi}{180}\right) = \frac{-3\pi}{2}$$
$$-540° = -540\left(\frac{\pi}{180}\right) = -3\pi$$

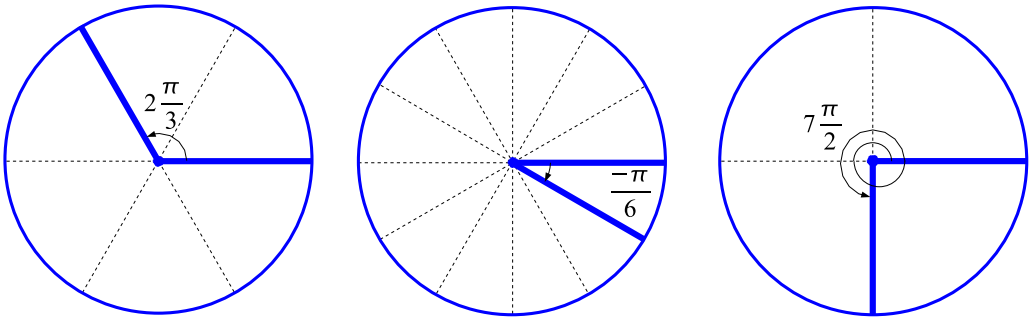
§8.1 #16 Construct angles with the following radian measure: $-\pi/4, -3\pi/2, -3\pi$.

A:



§8.1 #18 Construct angles with the following radian measure: $2\pi/3, -\pi/6, 7\pi/2$.

A:



§8.2 #24 Find t such that $0 \leq t \leq \pi$ and t satisfy $\cos t = \cos(-4\pi/6)$.

A: We use the following property of cosines:

$$\begin{aligned} \cos t &= \cos\left(\frac{-4\pi}{6}\right) \\ &= \cos\left(\frac{4\pi}{6}\right), \end{aligned}$$

Therefore $t = \frac{4\pi}{6} = \frac{2\pi}{3}$.

§8.2 #32 Find t such that $-\pi/2 \leq t \leq \pi/2$ and t satisfy $\sin t = -\sin(-\pi/3)$.

A: We use the following property of sines:

$$\begin{aligned} \sin t &= -\sin\left(\frac{-\pi}{3}\right) \\ &= \sin\left(\frac{\pi}{3}\right) \end{aligned}$$

Therefore $t = \frac{\pi}{3}$.

§8.2 #34 Find t such that $-\pi/2 \leq t \leq \pi/2$ and t satisfy $\sin t = -\cos t$.

A: Two different (but equally correct) methods can be used to solve this problem.

Method 1: We need to use trigonometric identities in order to rewrite $-\cos t$ in terms of sines. We know that $\cos t = \sin(\frac{\pi}{2} - t)$ or $\cos t = \sin(\frac{\pi}{2} + t)$. So we have 2 cases:

Case I: $\cos t = \sin(\frac{\pi}{2} - t)$

$$\begin{aligned}
\sin t &= -\cos t \\
\sin t &= -\sin\left(\frac{\pi}{2} - t\right) \\
\sin t &= \sin\left[-\left(\frac{\pi}{2} - t\right)\right] \\
\sin t &= \sin\left(t - \frac{\pi}{2}\right) \\
\sin t &= \sin\left(t - \frac{\pi}{2} + 2k\pi\right) \quad (k \text{ is an integer}) \\
t &= t - \frac{\pi}{2} + 2k\pi \\
0 &= -\frac{\pi}{2} + 2k\pi
\end{aligned}$$

This equation has no solutions. So we move on to Case II.

Case II: $\cos t = \sin(\frac{\pi}{2} + t)$

$$\begin{aligned}
\sin t &= -\cos t \\
\sin t &= -\sin\left(\frac{\pi}{2} + t\right) \\
\sin t &= \sin\left[-\left(\frac{\pi}{2} + t\right)\right] \\
\sin t &= \sin\left(-\frac{\pi}{2} - t\right) \\
t &= -\frac{\pi}{2} - t + 2k\pi \quad (k \text{ is an integer}) \\
2t &= -\frac{\pi}{2} + 2k\pi \\
t &= -\frac{\pi}{4} + k\pi \\
t &= -\frac{\pi}{4} + (0)\pi = -\frac{\pi}{4}
\end{aligned}$$

Method 2: Recall from Problem 33 that $t = \pi/4$ satisfies $\sin t = \cos t$. Using this information plus some identities of sine and cosine, we can say that

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$$

$$-\sin\left(-\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$$
$$-\sin\left(-\frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right).$$

From this, we conclude that $t = -\frac{\pi}{4}$.

§8.2 #38 Determine the value of $\cos t$ when $t = 5\pi, -2\pi, 17\pi/2, -13\pi/2$.

A: Use the identity $\cos t = \cos(t + 2\pi k)$, where k is an integer (positive or negative) chosen such that $0 \le t \le 2\pi$:

$$\cos(5\pi) = \cos(5\pi + 2\pi k) = \cos(5\pi - 2\pi(2)) = \cos(\pi) = -1$$

$$\cos(-2\pi) = \cos(-2\pi + 2\pi k) = \cos(-2\pi + 2\pi(1)) = \cos(0) = 1$$

$$\begin{aligned}\cos\left(\frac{17\pi}{2}\right) &= \cos\left(\frac{17\pi}{2} + 2\pi k\right) = \cos\left(\frac{17\pi}{2} + \frac{4\pi}{2}k\right) = \cos\left(\frac{17\pi}{2} + \frac{4\pi}{2}(4)\right) \\ &= \cos\left(\frac{\pi}{2}\right) = 0\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{-13\pi}{2}\right) &= \cos\left(\frac{-13\pi}{2} + 2\pi k\right) = \cos\left(\frac{-13\pi}{2} + \frac{4\pi}{2}k\right) = \cos\left(\frac{-13\pi}{2} + \frac{4\pi}{2}(3)\right) \\ &= \cos\left(\frac{-\pi}{2}\right) = 0\end{aligned}$$

§8.2 #40 Assume that $\sin(.42) = .41$. Use properties of cosine and sine to determine $\sin(-.42), \sin(6\pi - .42)$, and $\cos(.42)$.

A:

$$\sin(-.42) = -\sin(.42) = -.41$$

$$\begin{aligned}\sin(6\pi - .42) &= \sin(6\pi - .42 + 2\pi k) = \sin(6\pi - .42 - 6\pi) = \sin(-.42) \\ &= -\sin(.42) = -.41\end{aligned}$$

$$\begin{aligned}\sin^2(.42) + \cos^2(.42) &= 1 \\ \cos^2(.42) &= 1 - \sin^2(.42) \\ \cos^2(.42) &= 1 - (.41)^2 \\ \cos(.42) &= \pm\sqrt{1 - (.41)^2}\end{aligned}$$

We have two possible solutions. However, $0 \leq 0.42 \leq \frac{\pi}{2}$, which means the angle 0.42 radians lies in the first quadrant, which in turn makes $\cos(0.42)$ positive. Therefore $\cos(0.42) = \sqrt{1-(.41)^2}$.