21-112 Calculus II:
Homework #3 Solutions

Kevin C. Lee

September 24, 2004

1 6.5

Problem 10:
The formula to calculate amount with the interest compounded continuously is Pe^{rt} (ref. to page 280). Hence the average value of the money during the next 20 years is:

$$\frac{1}{20}\int_0^{20} 100e^{0.05t}\,dt = \frac{1}{20}(\frac{100}{0.05}e^{0.05t})|_0^{20} = 100(e^1 - 1) \approx \$172$$

Problem 24:
The formula to calculate *the future value of a continuous stream* is $\int_0^N Ke^{r(N-t)}\,dt$. Hence the balance after 6 years with compound interest 4.5% is:

$$\begin{aligned}\int_0^6 14000e^{0.045(6-t)}\,dt &= 14000(\frac{-e^{0.045(6-t)}}{0.045})|_0^6 \\ &= 14000(\frac{e^{0.27}-1}{0.045}) \\ &\approx \$96433\end{aligned}$$

Problem 26:

The formula to calculate *the future value of a continuous stream* is $\int_0^N Ke^{r(N-t)}\,dt$. Hence the balance after 10 years with compound interest 4.25% is:

$$\begin{aligned}
100000 &= \int_0^{10} Ke^{0.0425(10-t)}\,dt \\
100000 &= K(-\frac{e^{0.0425(10-t)}}{0.0425})|_0^{10} \\
100000 &= K(\frac{e^{0.425}-1}{0.0425}) \\
K &= \frac{100000 * 0.0425}{e^{0.425}-1} \\
K &\approx \$8025
\end{aligned}$$

Problem 30:
The volume is:

$$\begin{aligned}
\int_0^h \pi(kx)^2\,dx &= \pi k^2 \int_0^h x^2\,dx \\
&= \pi k^2(x^3/3)|_0^h \\
&= \pi k^2(h^3/3) \\
&= (\pi k^2 h^3)/3
\end{aligned}$$

Problem 38:
We know that $\triangle x = \frac{b-a}{n}$. Since each term is multiplied by 0.5, $\triangle x = 0.5$. Furthermore, there are 6 terms so $n = 6$. Given $\triangle x = 0.5$, $n = 6$, and $a = 1$, we know that $b = n*\triangle x+a = 6 * 0.5 + 1 = 4$.

Observe that what varies in each term within the bracket is the denominator and its increment is $\triangle x$. Hence, it must be our x variable. Therefore, $f(x) = 3/x$.

Problem 42:
Observe that what varies within the bracket in each term is what is inside the parentheses and their increment is $\triangle x$. Hence, it must be our x variable. Therefore, $f(x) = 2x + x^3$.

The area under $f(x)$ from $0 \leq x \leq 1$ is:

$$\begin{aligned}\int_0^1 (2x + x^3)\, dt &= (x^2 + x^4/4)|_0^1 \\ &= 1 + 1/4 \\ &= 5/4\end{aligned}$$