21-112 Calculus II:
Homework #2 Solutions

Kevin C. Lee

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1 6.3

Problem 38:
It is the total amount of pollutants in tons that are discharged from 1995 to 1997.

Problem 42:
(a) The total profit is simply the integral of the marginal profit function of $100+50x-3x^2$ from 5 to 8. Solving the integral, we get:

$$\begin{aligned}\int_5^8 (100 + 50x - 3x^2)\,dx &= (100x + 25x^2 - x^3)|_5^8 \\ &= (100 * 8 + 25 * 8^2 - 8^3) - (100 * 5 + 25 * 5^2 - 5^3) \\ &= 888\end{aligned}$$

(b) It is the area under the curve of $100 + 50x - 3x^2$ from 5 to 8.

Problem 45:
(a) The area under the graph of $y = r(t)$ over the interval $0 \leq t \leq 2$ is:

$$\begin{aligned}\int_0^2 (12 + \frac{4}{(t+3)^2}\, dt &= (12t - \frac{4}{(t+3)})|_0^2 \\ &= (12 * 2 - \frac{4}{(2+3)}) - (0 - \frac{4}{(0+3)}) \\ &= 24 - 4/5 + 4/3 \\ &= 368/15\end{aligned}$$

(b) The total temperature fall in 2 hours.

2 6.4

Problem 22:

We need to break the area into two regions since the two regions are not bounded by the same function from above. The boundaries of the regions can be solved by computing intersection points. The first intersection point happens when $x = 1/2$ by setting $1/x = 4x$. The second intersection point happens when $x = \sqrt{2}$ by setting $1/x = x/2$. Hence the area is:

$$\begin{aligned}\int_0^{1/2} (4x - x/2)\, dx + \int_{1/2}^{\sqrt{2}} (1/x - x/2)\, dx &= (2x^2 - x^2/4)|_0^{1/2} + (\ln|x| - x^2/4)|_{1/2}^{\sqrt{2}} \\ &= 7/4 * 1/4 + \ln\sqrt{2} - 1/2 - \ln 1/2 + 1/16 \\ &= \ln 2\sqrt{2}\end{aligned}$$

Problem 24:

Similar to Problem 22, we need to break the area into two regions since the two regions are not bounded by the same function from above. The boundaries of the regions can be solved by computing intersection points. The first intersection point happens when $x = 2$ by setting $4x = 12 - x^2$. Notice that we get two x's and we only take the positive value because we are only considering the boundary points on the first quadrant. The second intersection point happens when $x = 3$ by setting $x = 12 - x^2$. Hence the area is:

$$\begin{aligned}\int_0^2 (4x - x)\,dx + \int_2^3 (12 - x^2 - x)\,dx &= (\frac{3x^2}{2})|_0^2 + (-\frac{x^3}{3} - \frac{x^2}{2} + 12x)|_2^3 \\ &= 6 + (-9 - 9/2 + 36 + 8/3 + 2 - 24) \\ &= 55/6\end{aligned}$$

Problem 28:
It is the difference of the distances between the two rockets.