21-112 Calculus II:
Homework #1 Solutions

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1 6.1

Problem 46:
(a)

$$s(t) = \int (-32t)\, dt = -16t^2 + C$$

Since we know at $t = 0$, $s(t) = 400$, substitute $t = 0$ into $s(t)$, we get:

$$s(0) = C = 400$$

Therefore,

$$s(t) = -16t^2 + 400$$

(b)
The rock reaches the ground when $s(t) = 0$. Plug it into the equation of $s(t)$, we get:

$$\begin{aligned} s(t) = -16t^2 + 400 &= 0 \\ -16t^2 &= -400 \\ t^2 &= 25 \\ t &= 5 \end{aligned}$$

Notice that we only take the positive answer (+5) as time can never be negative.
(c)

The velocity when the rock reaches the ground happens when t = 5. Plug it into $v(t)$, we get:

$$v(5) = -32 * 5 = -160 \ ft/sec$$

The negative sign is due to the fact that the rock is going downward.

Problem 48:

The total mine after t hours of operation is the integral of the function given. Integrate the function, we get:

$$\int (40 + 2t - \frac{1}{5}t^2)\, dt = 40t + t^2 - \frac{1}{15}t^3 + C$$

Since at $t = 0$, there is nothing produced, $C = 0$. Therefore, the function is:

$$Total(t) = 40t + t^2 - \frac{1}{15}t^3$$

Problem 52:

Again we take the integral of the function, we get:

$$Total(x) = \int (0.2x + 1)\, dx = 0.1x^2 + x + C$$

Solve for C when $x = 0$ and $Total(0) = 2$.

$$Total(0) = C = 2,$$
$$Total(x) = 0.1x^2 + x + 2$$

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Problem 7:

$$\triangle x = \frac{3 - 1}{5} = 0.4$$

So the subintervals are [1, 1.4], [1.4, 1.8], [1.8, 2.2], [2.2, 2.6], and [2.6, 3], where the first number of each bracket is the left endpoint and the second number is the right endpoint on the x-axis.

Since we are using the left endpoints, our area is:

$$\begin{aligned} area &\approx 0.4(f(1) + f(1.4) + f(1.8) + f(2.2) + f(2.6)) \\ &\approx 0.4(1^3 + 1.4^3 + 1.8^3 + 2.2^3 + 2.6^3) \\ &\approx 15.12 \end{aligned}$$

Problem 8:

$$\triangle x = \frac{1-0}{5} = 0.2$$

So the subintervals are [0, 0.2], [0.2, 0.4], [0.4, 0.6], [0.6, 0.8], and [0.8, 1.0] where the first number of each bracket is the left endpoint and the second number is the right endpoint on the x-axis.

Since we are using the right endpoints, our area is:

$$\begin{aligned} area &\approx 0.2(f(0.2) + f(0.4) + f(0.6) + f(0.8) + f(1.0)) \\ &\approx 0.2(0.2^3 + 0.4^3 + 0.6^3 + 0.8^3 + 1.0^3) \\ &\approx 0.36 \end{aligned}$$

Problem 17:

$$\triangle x = \frac{1-(-1)}{5} = 0.4$$

So the subintervals are [-1, -0.6], [-0.6, -0.2], [-0.2, 0.2], [0.2, 0.6], and [0.6, 1.0] where the first number of each bracket is the left endpoint and the second number is the right endpoint on the x-axis.

Since we are using the *midpoints*, our area is:

$$\begin{aligned} area &\approx 0.4(f(-0.8) + f(-0.4) + f(0) + f(0.4) + f(0.8)) \\ &\approx 0.4(\sqrt{1-(-0.8)^2} + \sqrt{1-(-0.4)^2} + \sqrt{1-0^2} + \sqrt{1-0.4^2} + \sqrt{1-0.8^2}) \\ &\approx 1.61321 \end{aligned}$$

Hence the error is 0.04241 $(= 1.61321 - 1.57080)$.

Problem 18:
(a)

$$\triangle x = \frac{1-0}{5} = 0.2$$

So the subintervals are [0, 0.2], [0.2, 0.4], [0.4, 0.6], [0.6, 0.8], and [0.8, 1.0] where the first number of each bracket is the left endpoint and the second number is the right endpoint on the x-axis.

Since we are using the *midpoints*, our area is:

$$\begin{aligned} area &\approx 0.2(f(0.1) + f(0.3) + f(0.5) + f(0.7) + f(0.9)) \\ &\approx 0.2(0.1^3 + 0.3^3 + 0.5^3 + 0.7^3 + 0.9^3) \\ &\approx 0.79300 \end{aligned}$$

Hence the error is 0.00760 $(= 0.79300 - 0.78540)$.
(b)
The area approximated in **(a)** is a quarter of a circle. Half circle is the double of it. So it is 1.58600. The error is 0.0152 $(= 1.58600 - 1.57080)$. It is less than the error computed in question **(17)** and therefore more accurate.