Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 21

Name:_____

Problem	Points	Score
1	35	
2	35	
3	30	
Total	100	

Q1: (35pts)
Find an expression for the total cost per period for the following inventory system. If you order an amount Q, it arrives immediately and the cost of the order is AQ^{α} for some $0 < \alpha < 1$. The inventory cost is I times M^{β} per period, for some $\beta > 0$, where M is the average inventory. The demand is λ units per period and there is a penalty cost of P time N^{γ} per period, where N is the average amount out of stock.

Q2: (35pts)
Players A and B play the following game. A chooses a number $x_A \in \{0, 1, 2, 3\}$ and B chooses a number $x_B \in \{0, 1, 2\}$. If $x_A + x_B$ is odd, A wins a point, otherwise B wins a point.
Write down a linear program whose solution will produce an optimum strategy for A. **YOU DO NOT HAVE TO SOLVE THE PROGRAM**.

Q3: (30pts)

There are two machines available for the processing of n jobs. The processing time of job j is $p_j > 0$ for $j = 1, 2, \ldots, n$. The objective is to assign jobs to machines in order to minimise $C_{\max} = \max\{C_j : j = 1, 2, \ldots, n\}$ where C_j is the completion time of job j. Let

$$P_1 = \sum_{j=1}^{n} p_j \text{ and } P_2 = \max\{p_j : j = 1, 2, \ldots, n\}$$

Show that the optimal solution satisfies

$$\frac{P_1}{2} \leq C_{\max} \leq \frac{P_1 + P_2}{2}.$$