Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 2

Name:_____

Problem	Points	Score
1	35	
2	35	
3	30	
Total	100	

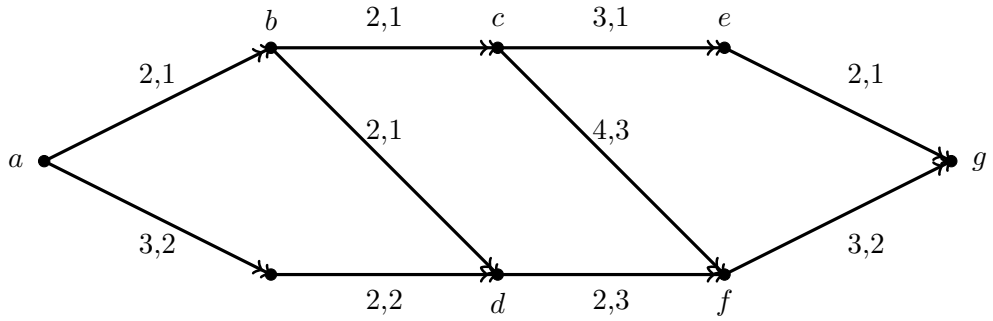
Q1: (35pts)

Two players simultaneously choose an integer between 1 and $n \geq 3$ inclusive. if the numbers are equal there is no payoff. If a player chooses a number one larger than the one chosen by their opponent then they win one. If a player chooses a number two or more larger than their opponent then they lose two.

Using dominance, reduce the game to a 3×3 game. What is the value of the reduced game?

(Hint: argue that pure strategy 1 dominates pure strategy $i \geq 4$.)

Q2: (35pts)
Find a minimum weight path from a to all other nodes of the digraph below. The 2 numbers x, y on a directed edge vw are to be interpreted as follows: If a path P from a to v has weight t, then the weight of the path $P + vw$ is $t + (x + yt)$. Assume that $t = 0$ at the start a.



Q3: (30pts)

There are two machines available for the processing of n jobs. The processing time of job j is $p_j > 0$ for $j = 1, 2, \ldots, n$. The objective is to assign jobs to machines in order to minimise $C_{\max} = \max\{C_j : j = 1, 2, \ldots, n\}$ where C_j is the completion time of job j. Let

$$P_1 = \sum_{j=1}^{n} p_j \text{ and } P_2 = \max\{p_j : j = 1, 2, \ldots, n\}$$

Show that the optimal solution satisfies

$$\frac{P_1}{2} \leq C_{\max} \leq \frac{P_1 + P_2}{2}.$$